

The Evolution and Ecology of AT Models: a partial survey

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Outline and vocabulary

- Early models
- Current ecology of Adaptive Therapy modeling
- Future directions

Focus on containment treatments.

- ◇ Continuous AT / containment : stabilizes tumor at a certain size
- ◇ Intermittent AT : stabilizes between two thresholds, on-off treatment
- ◇ Maximal Tolerated Dose (MTD)

Simple models : sensitive and resistant tumor cells + single drug.

(N.B : throughout, the notation differs from original articles)

2 types of tumor cells : S sensitive, R fully resistant, $N = S + R$

$$\dot{R} = g(N)(R + \mu S), \quad \text{with } g(N) \text{ decreasing, } \mu \text{ mutation}$$

Goal : Maximize survival time (time at which $N > N_{crit}$)

Assumption : $S(t)$ perfectly tunable, chosen to minimize $\dot{R}(t)$

Trade-off : S large maximizes competition, but also mutations.

- $R < \bar{R} \rightarrow S$ small (minimize mutations) *
- $R > \bar{R} \rightarrow S$ large (maximize competition) *

* This is what is intuitively expected, and what Hansen et al. (2017) show in specific cases, but not true for all functions $g(N)$.

3 special cases

- 1 Exponential growth : $g(N) = \rho \rightarrow$ MTD slightly better
- 2 Logistic : $g(N) = \rho(1 - N/K) \rightarrow$ MTD and containment comparable
- 3 Gompertz : $g(N) = \rho \ln(K/N) \rightarrow$ containment much better.

Hansen et al. (2017) : pedagogical treatment of (mostly) logistic case
+ add various types of resistance costs + resistance costs not needed.

V. & Noble (2020) : impact of ongoing mutations may be quantified,
very small effect on survival time.

H. Monro and E. Gaffney (2009)

Simulations of Gompertzian model with Norton-Simon kill-rate :

$$\begin{array}{ll} \text{sensitive} & \dot{S} = \rho \ln(K/N)(1 - \lambda C - \mu)S \\ \text{resistant} & \dot{R} = \rho \ln(K/N)(R + \mu S) \end{array}$$

with K carrying capacity, λ sensitivity, C dose, μ mutation.

Questions : best constant dose ? best treatment starting time ?

Results : moderate dose + delaying treatment increases survival time.

- very high dose \rightarrow death from resistant cells
- very low dose \rightarrow death from sensitive cells

\leftrightarrow Optimal dose strikes a balance.

Moreover, delaying treatment increases competition.

Coined the word + different spirit + first paper with preclinical data

A mysterious model + **first frequency-dependent model**, similar to :

$$\dot{N}_i / N_i = \rho \frac{w_i}{\bar{w}} - \lambda_i C e$$

- N_i : number of cells of type i
- $\bar{w} = \sum_i \frac{N_i}{N} w_i$ average fitness parameter ;
- $e(t)$ environmental sensitivity (dynamics unshown)

Simulations with 5 types : fittest but most sensitive, less fit & sensitive, less fit & resistant, environmentally resistant, fittest & resistant.

↔ AT improves on MTD (unless fittest & resistant type dominant)

Discrete-time, frequency dependent model similar to :

$$\dot{S}/S = \rho_s(\text{other treatment}) \frac{S}{S+R} - \lambda_s C \simeq \rho_s - \lambda_s C \quad \text{if } R \ll S$$

$$\dot{R}/R = \rho_r(\text{other treatment}) \frac{R}{S+R} - \lambda_r C \simeq -\lambda_r C \quad \text{if } R \ll S$$

λ_s, λ_r sensitivity to treatment ; ρ_s, ρ_r affected by auxiliary treatment

↔ resistance cost may be increased to improve AT.

Issue : basic model too favorable to AT.

Bacevic & Noble et al. (2017) : replace $\frac{R}{S+R}$ by $f(R)$ with $f(0) > 0$.

↔ key-parameter is $f(0)$: relative fitness of resistant cells when rare.

Three types Lotka-Volterra model : 2 sensitive + 1 fully resistant

$$\frac{\dot{N}_i}{N_i} = \rho_i \left(1 - \frac{\sum_{j=1}^3 \alpha_{ij} N_j}{K_i(\text{treatment})} \right)$$

N_i : cells of type i , K_i carrying capacity, α_{ij} competition-coefficient.

Whether there is a cost of resistance is debatable (I would say no)

Compare Intermittent AT to MTD and metronomic via simulations.

Famous as in [paper reporting results of first clinical trial](#)

But other models would have generated similar motivation

Moreover, somewhat complex and imprecise, and led to confusions.

Other Lotka-Volterra models

$$\begin{aligned}\text{Carrère (2017)} : \dot{S}/S &= \rho \left(1 - \frac{R+S}{K} \right) - \lambda C \\ \dot{R}/R &= \rho \left(1 - \frac{R+\alpha S}{K} \right), \quad \alpha > 1\end{aligned}$$

Standard treatment impact, α potentially large (cost of resistance + indefinite containment), and not simulations but **optimal control**.

Carrère and Zidani (2020) : **uncertainty** on parameters + staying below a maximal acceptable tumor size.

Pouchol et al (2018) : **infinite number of types**, two types of drugs.

Strobl et al. (2020) : birth-death model, **effect of cell turnover**.

No PDE models applied to AT, but some models and simulations :

Bacevic & Noble et al. (2017), Gallaher et al. (2018), Li et al. (2017)

Space may be important as :

- boundary growth may justify assumption of competition
- if spatial separation between cells types, low competition
- if resistant cells trapped in tumor core, increases resistance cost
- impacts comparison between continuous AT, intermittent AT, MTD

↔ more tomorrow !

- Evolutionary games including normal cells : West et al. (2018), Kaznatcheev et al. (2019),...
- More optimal control models : Gluzman et al. (2020), Ledzewicz & Schättler (2019),...
- Leader-Follower games (Stankova et al., 2019)
- ...

Current ecology of Adaptive Therapy modeling

Still in Moffitt's gravity field (+ Moffitt's metastases), but expanding

Different styles :

- Simulations of potentially complex models (e.g., Moffitt)
- More or less hard-core optimal control
- Evolutionary game theory models
- Simple maths for pedagogical purposes
- People I do not know very well...

↔ need to hear non-Moffitt voices + building a community

Some personal messages

To simulators : **theoreticians may help** to make sense of your results

To hard-core optimal controllers or strong mathematicians :

- Most of us are not able to follow, please translate!
- Finding optimal treatment is great, but **comparing containment to MTD is already interesting...**
- ... and then easier to vary assumptions to stress-test conclusions.

To theoreticians like me who try to avoid simulations : you're wrong!

What do we need to make better models? - I

- 1 Talking, putting our abilities together
- 2 More data : e.g., please, make Zhang et al. data available!
- 3 Be precise : e.g., in density-dependent models, are our variables local densities or total population sizes? May be crucial!
- 4 Space : PDE models?
- 5 Trade-offs between benefits of AT and potential downsides : current models explain why AT could work, not whether AT is likely to work!
- 6 Risk of adverse event due to large tumor burden (Mistry, 2020)

What do we need to make better models? - II

- 1 Partially resistant cells : then not clear that AT superior to MTD
- 2 More than two sensitivity levels + mutations : $S_1 \rightarrow S_2 \rightarrow R$
- 3 Driver mutations, drug-induced resistance
- 4 Multidrug AT
- 5 Includes normal cells, immune system...
- 6 Specificities of prostate cancer ?

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