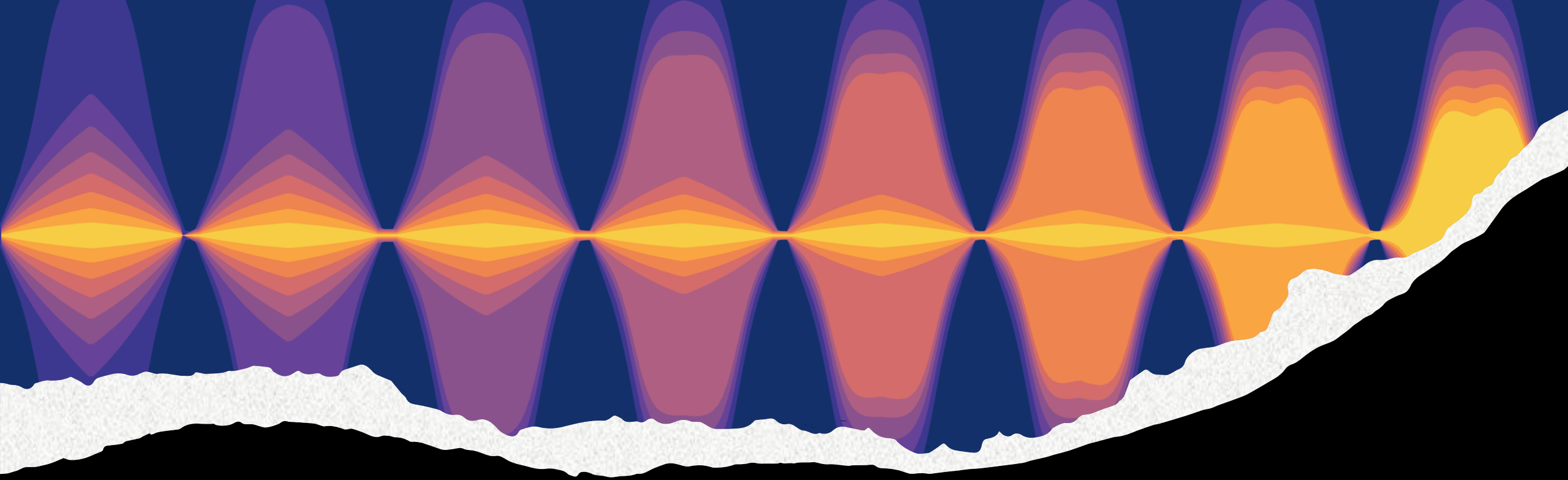




Towards multi-drug adaptive therapy

Jeffrey West (Moffitt Cancer Center)

CATMo2020

Towards multi-drug adaptive therapy



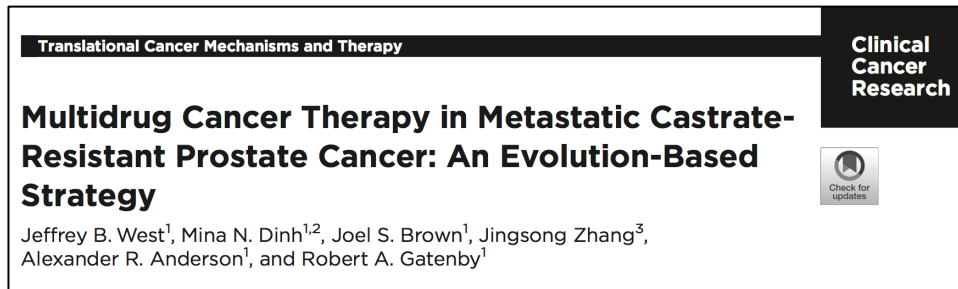
1.



2.



3.



4.



Clinically feasible today

Conceptual ideas for tomorrow



1

2

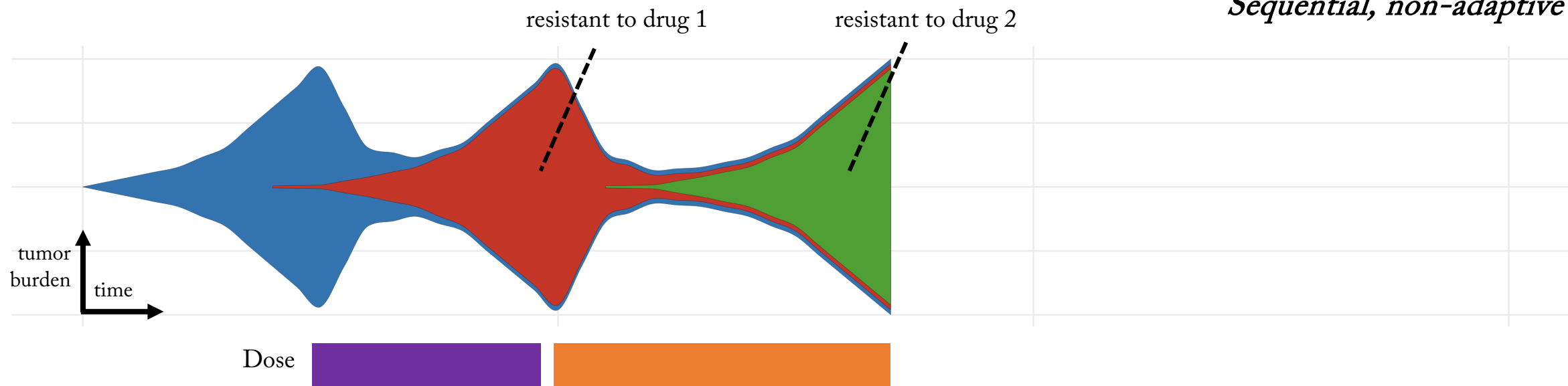
3

4

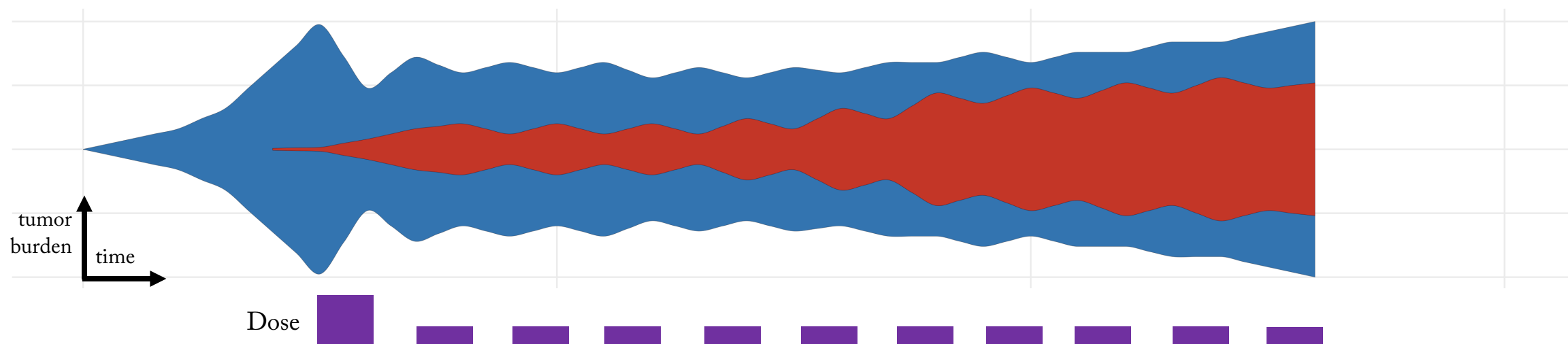
Sequential and Adaptive Therapy



Sequential, non-adaptive



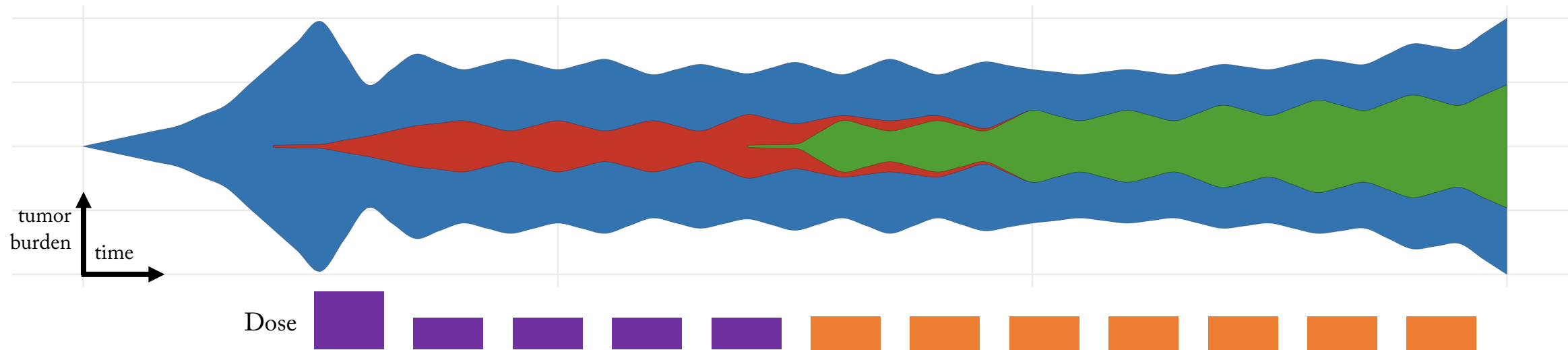
Single drug adaptive



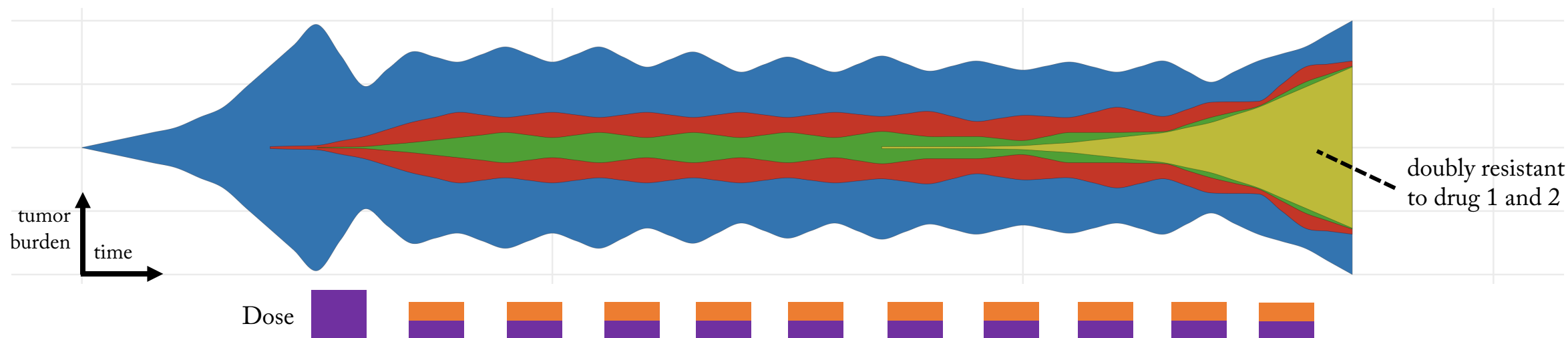
Designing Adaptive Therapy schedules w/ multiple drugs



Two drug combination adaptive



Two drug sequential adaptive

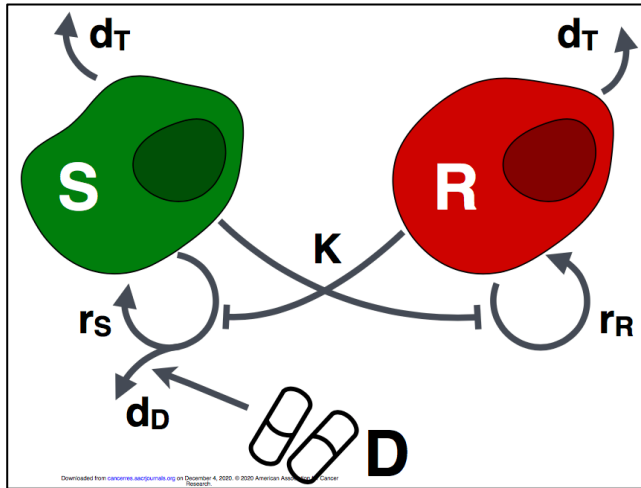


What is the purpose of adding a second drug?



The model:

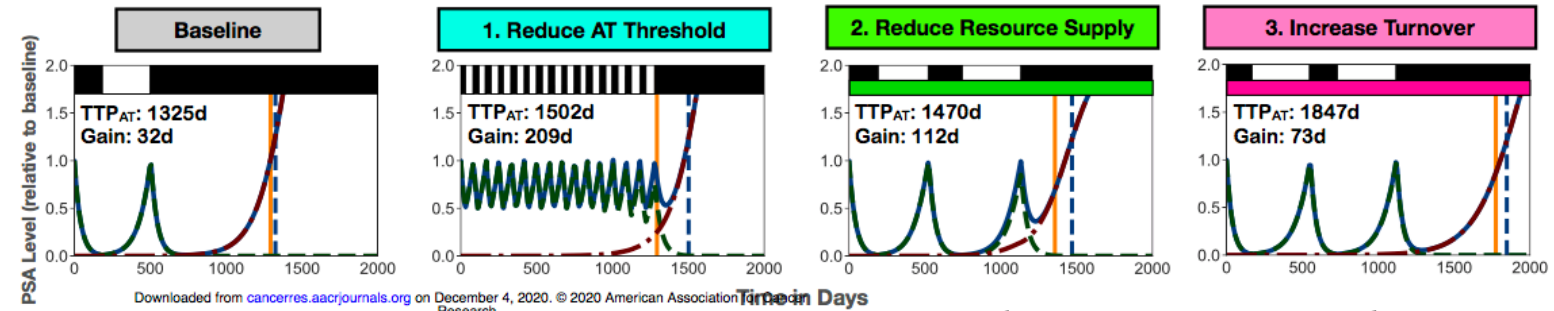
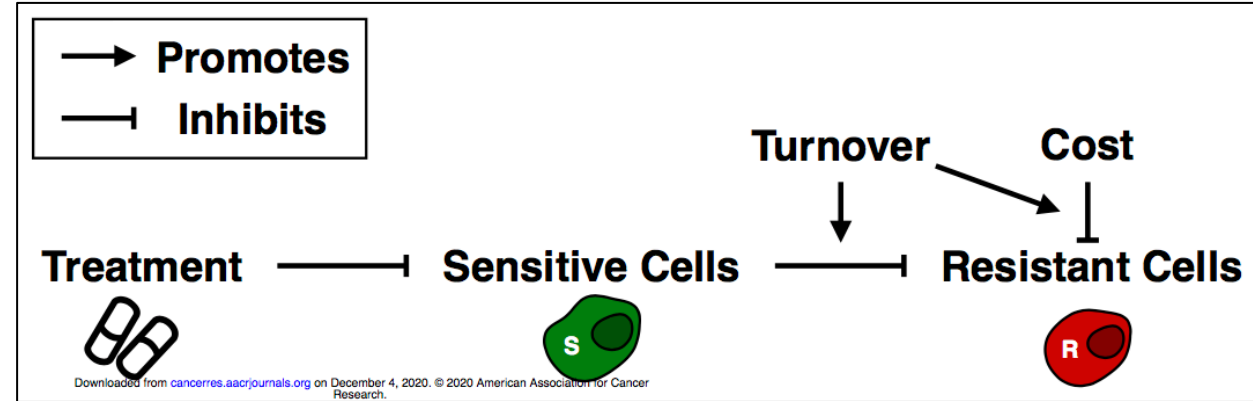
- Lotka-Volterra competition
- Norton-Simon drug effect



$$\frac{dS}{dt} = r_S \left(1 - \frac{S+R}{K} \right) \left(1 - \frac{2d_D}{D_{Max}} D(t) \right) S - d_T S,$$

$$\frac{dR}{dt} = r_R \left(1 - \frac{R+S}{K} \right) R - d_T R,$$

(Strobl, et. al. Cancer Research 2020)



Idea:

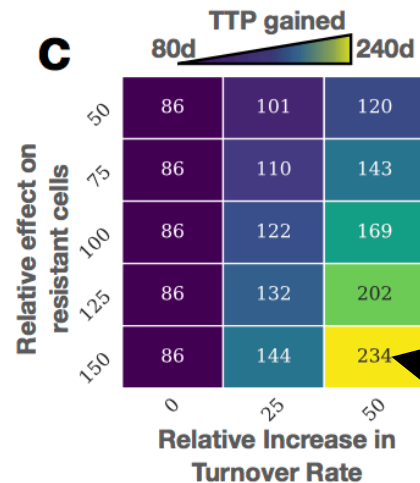
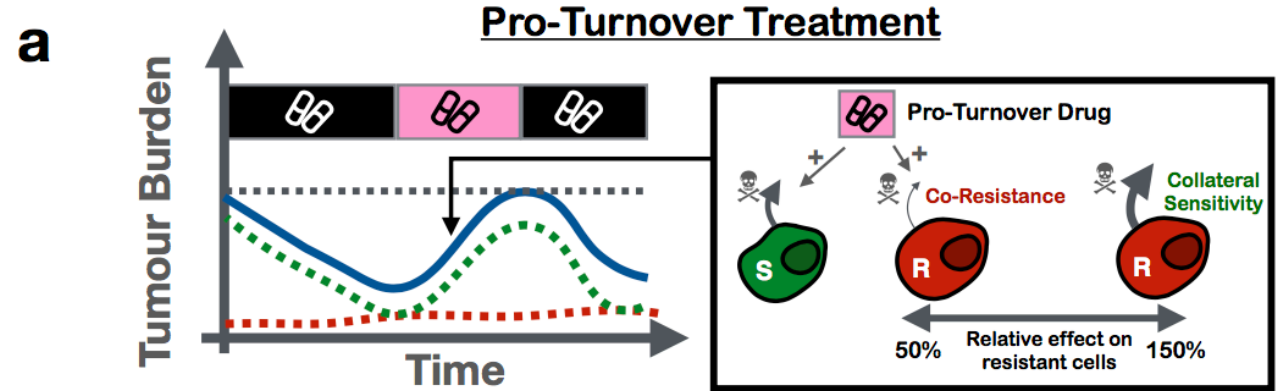
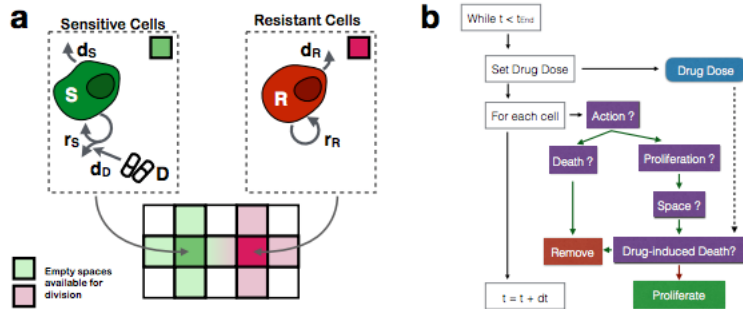
- Targeting shared resources or increasing turnover increases benefit of single drug Adaptive Therapy

Pro-turnover treatment (off treatment cycle)



The model:

- Agent-based 2-d analog of Lotka-Volterra model



Idea:

- Turnover increases competition – can we increase competition during off treatment periods?

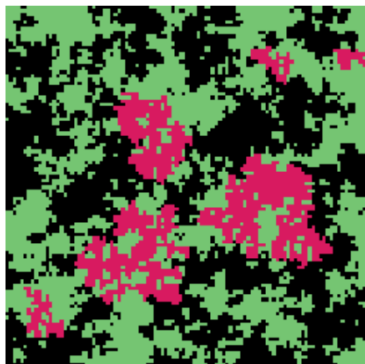
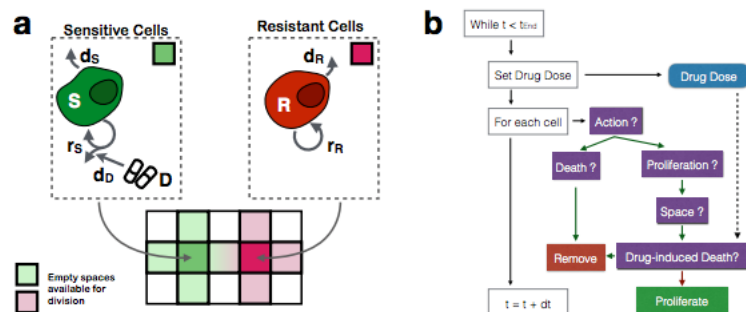
Increasing turnover, targeted to resistant cells

Pro-proliferation treatment (off treatment cycle)



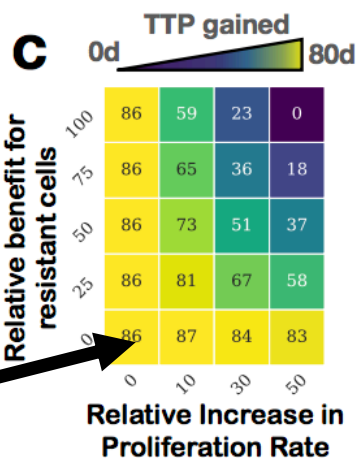
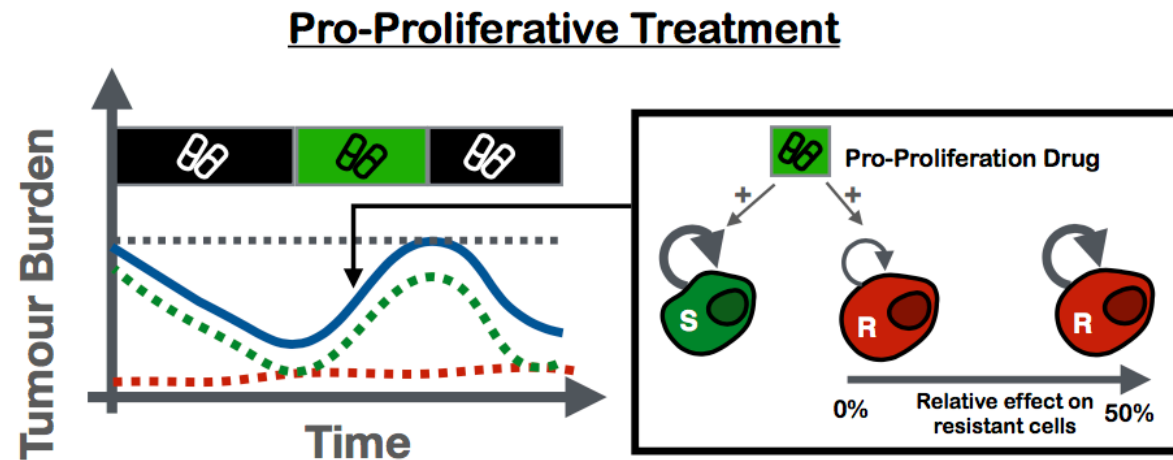
The model:

- Agent-based 2-d analog of Lotka-Volterra model



Optimal is to leave well-enough alone!

a



Idea:

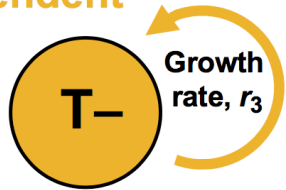
- Sensitive cells suppress Resistant -- can we boost competition by increasing proliferation off treatment?

1. Primary-secondary adaptive therapy

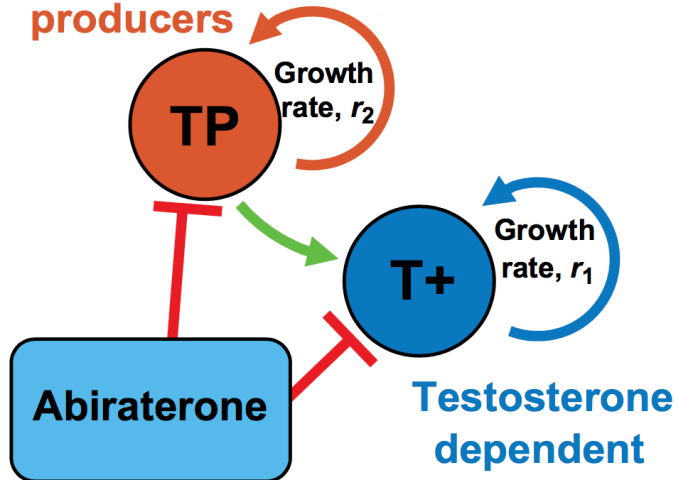
"What's the difference between theory and practice? Small in theory; large in practice."

- Author attribution unknown -

Testosterone
independent



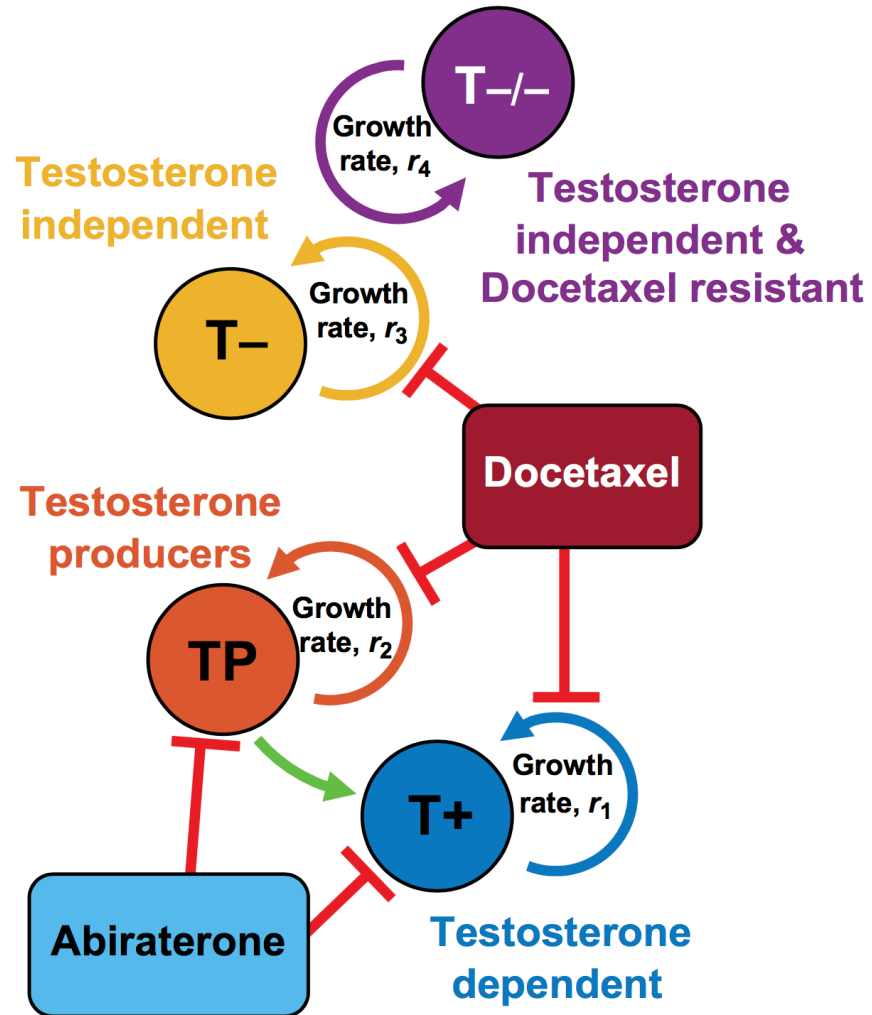
Testosterone
producers



Cell Populations:

- Testosterone **producers** (TP)
- Testosterone **dependent** (T+)
- Testosterone **independent**

$$\dot{y}_i = r_i y_i \left(1 - \frac{\sum_{j=1}^3 a_{ij} y_j}{K_i} \right)$$



Cell Populations:

- Testosterone **producers** (TP)
 - Testosterone **dependent** (T+)
 - Testosterone **independent**
 - **Susceptible** to Docetaxel
 - **Resistant** to Docetaxel
- Targeted by "Primary" drug
- Targeted by "Secondary" drug

Primary drug

- Greatest efficacy and/or lower toxicity

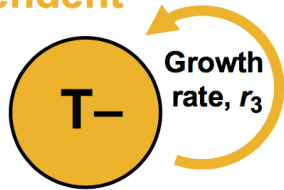
Secondary drug

- Targets cell population which is resistant to primary drug

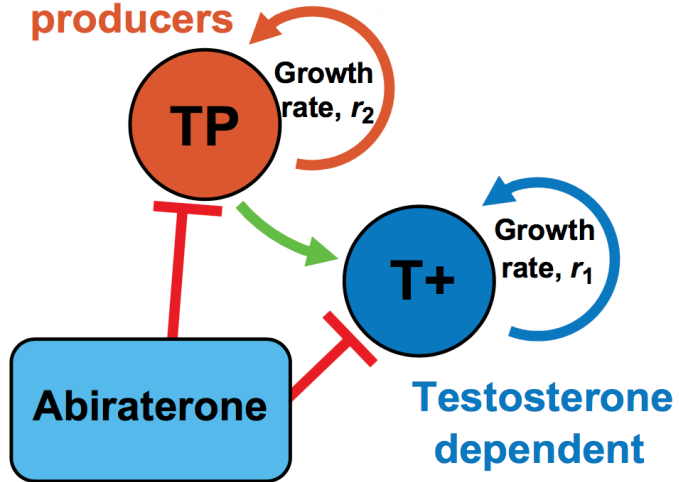
Primary mono-therapy



Testosterone independent

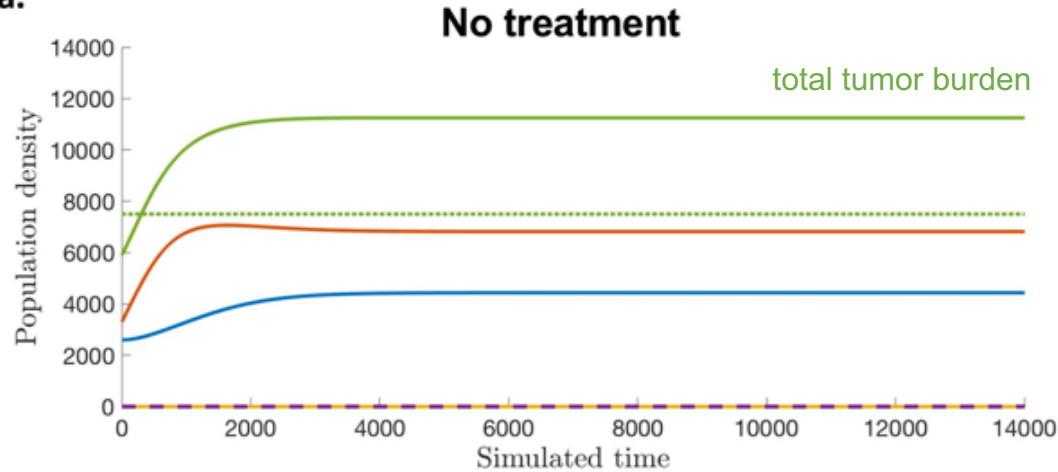


Testosterone producers

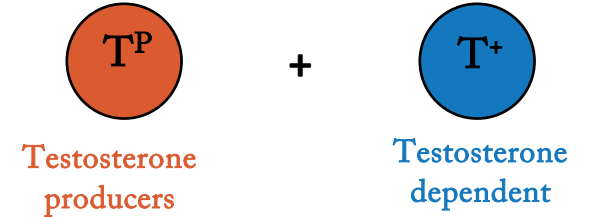


(Zhang. et. al. Nature Comm. 2017)

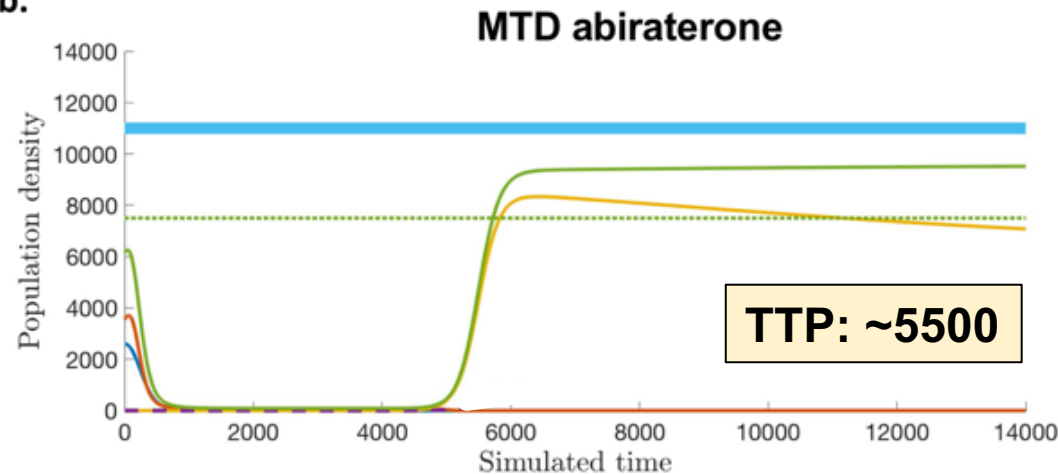
a.



Co-existence



b.



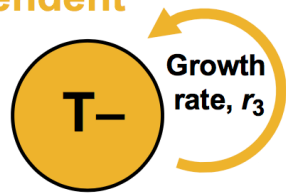
Relapse dominated by resistance



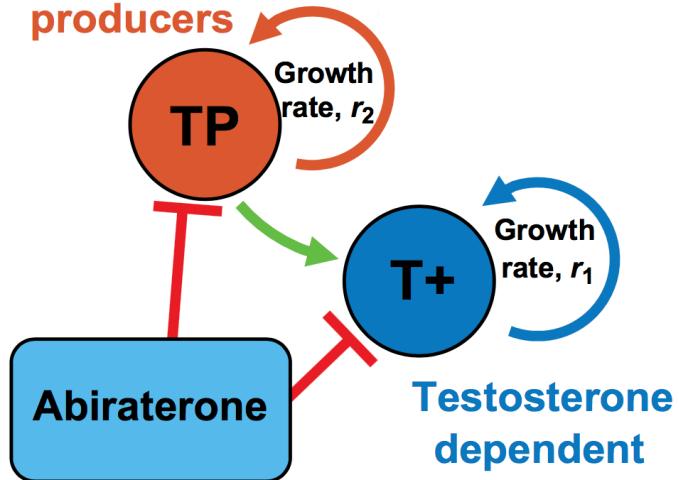
Primary mono-therapy (adaptive)



Testosterone independent

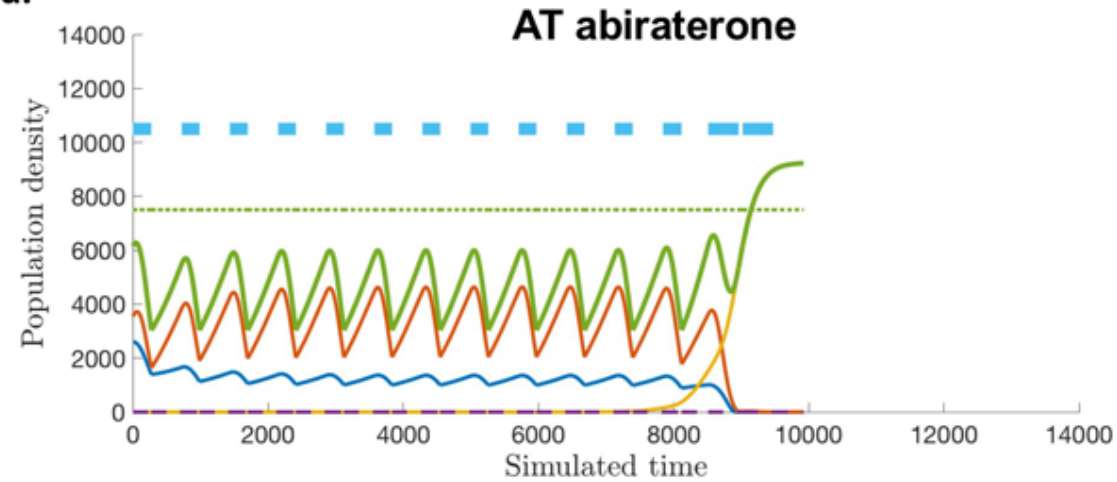


Testosterone producers

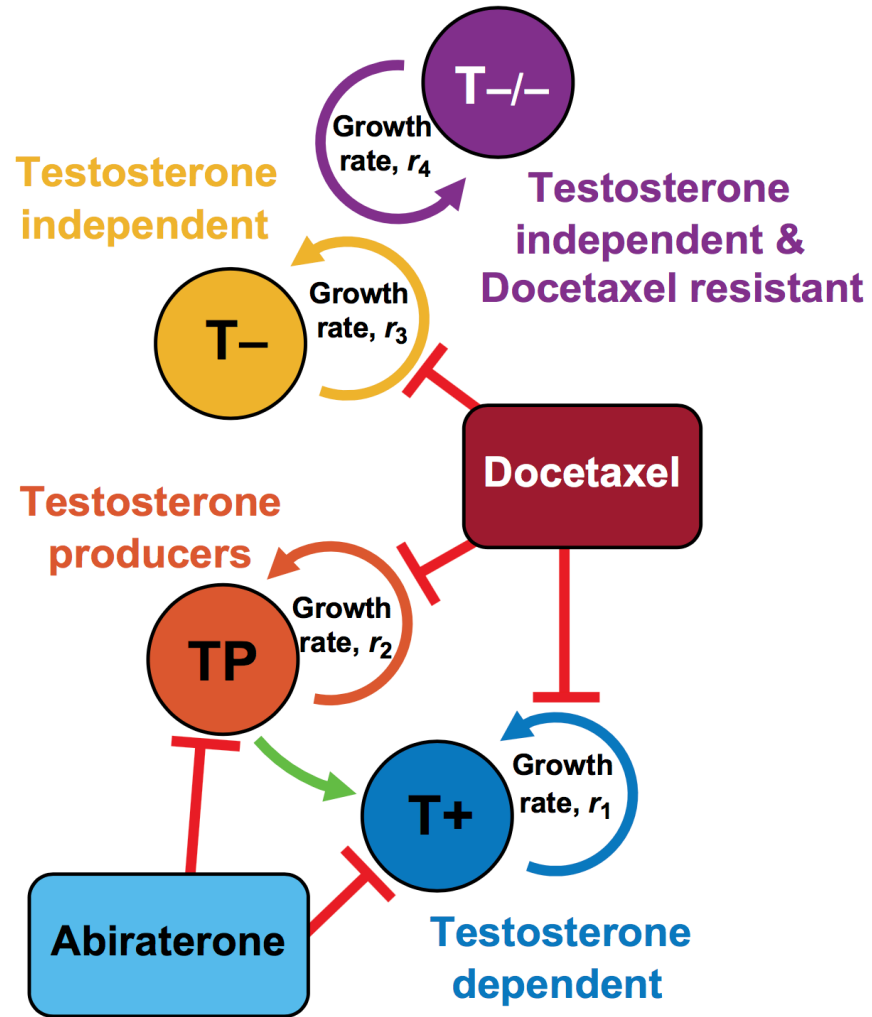


Testosterone dependent

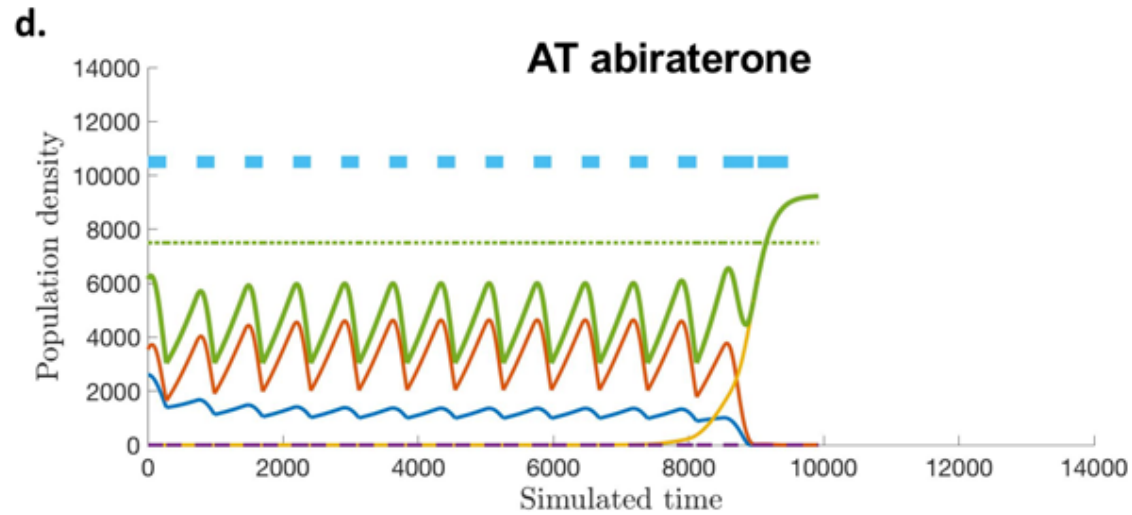
d.



Can docetaxel be used as “Secondary” drug?



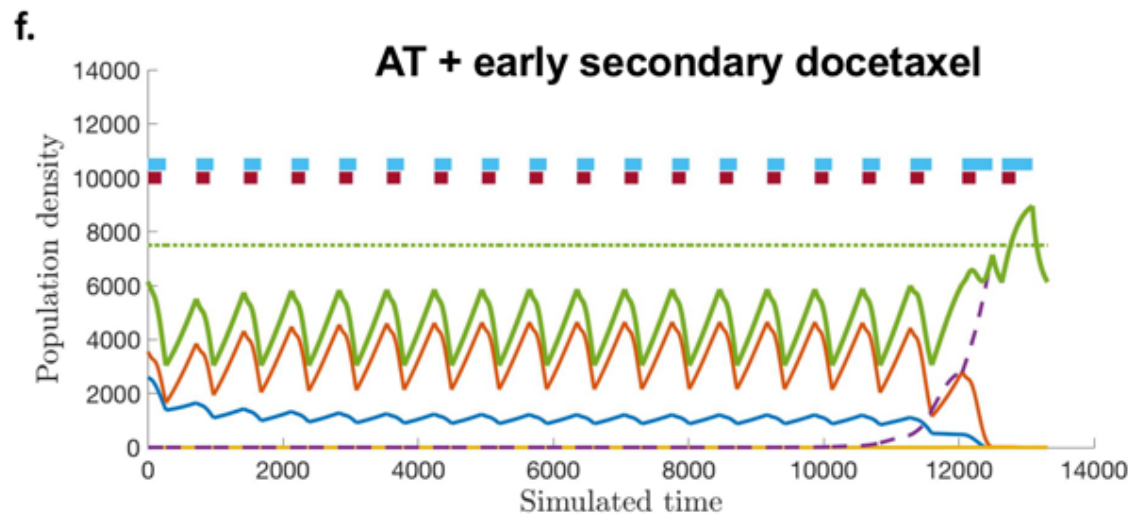
(Zhang. et. al. Nature Comm. 2017)



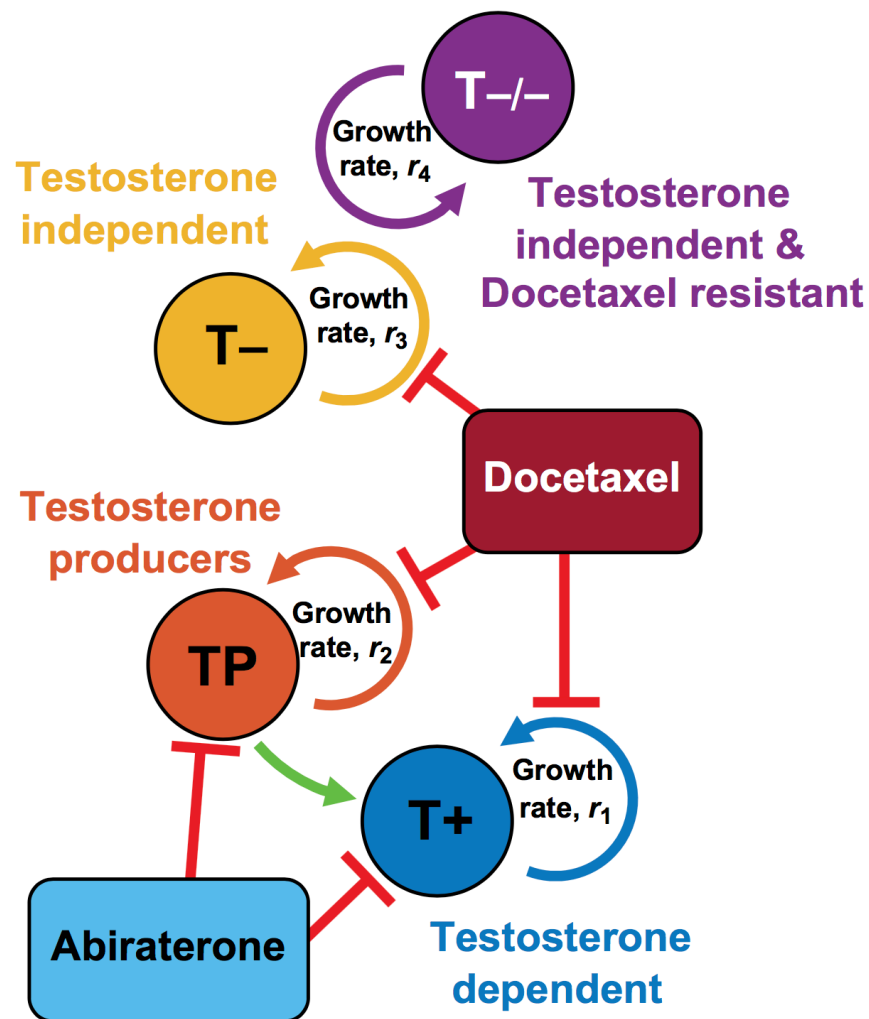
$TTP_{CT}: \sim 5500$

$TTP_{AT}: \sim 9500$

$TTP_{PS}: \sim 13000$



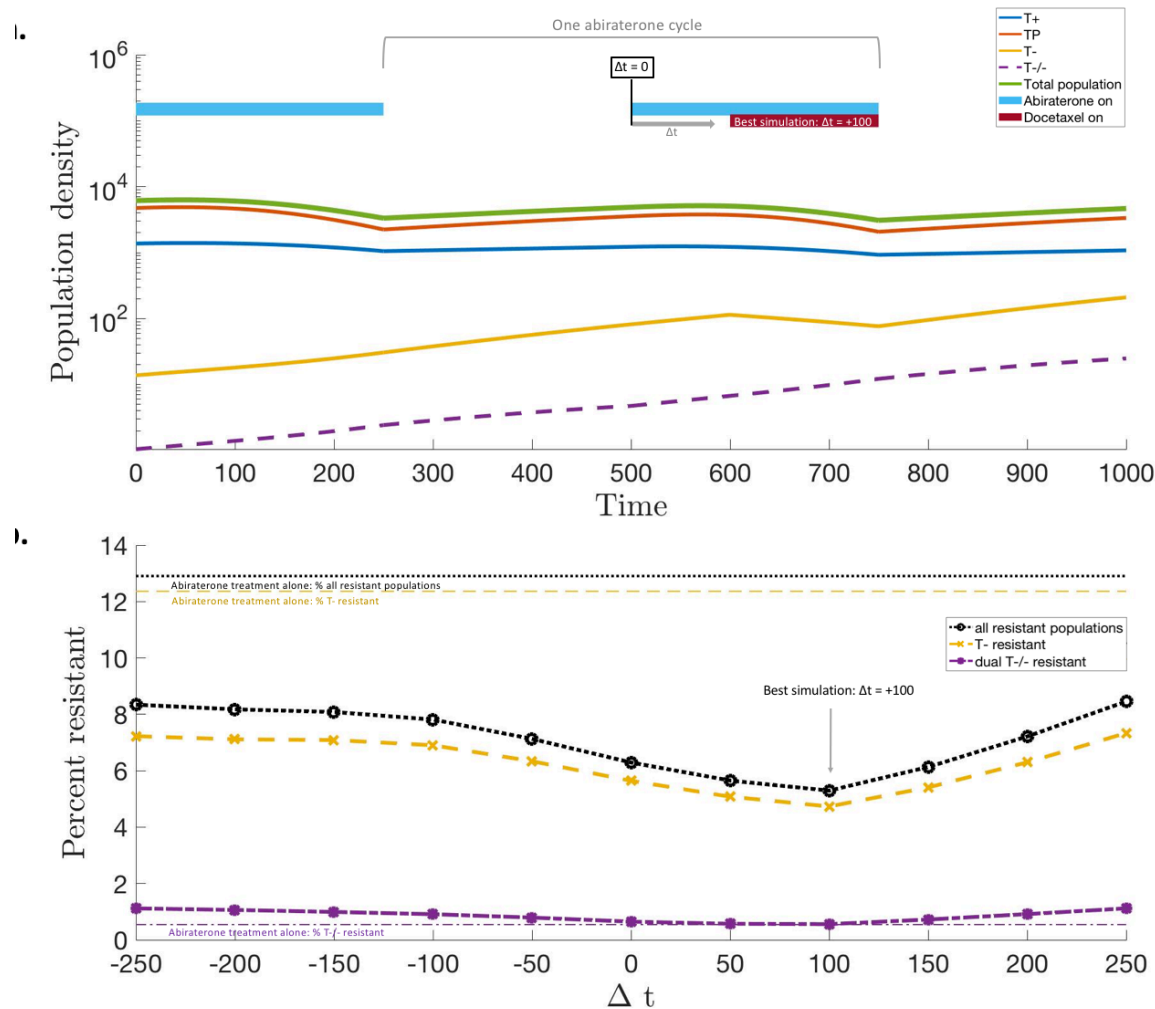
Can docetaxel be used as “Secondary” drug?



(Zhang. et. al. Nature Comm. 2017)

Problem with our secondary drug:

- Docetaxel is indiscriminately targeting 3 cell types



Delayed docetaxel:

- resistant cell type (T-) is growing
- sensitive cell types (TP, T+) are decaying

2. Evolutionary cycles

" Much is known but unfortunately in different heads."

- Werner Kollath -

Population Dynamics Lotka-Volterra

$$\dot{y}_i = r_i y_i \left(1 - \frac{\sum_{j=1}^3 a_{ij} y_j}{K_i} \right)$$

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Frequency Dynamics Replicator Dynamics

$$\dot{x}_i = (f_i - \phi) x_i$$

$$f_i = w_i (A \vec{x})_i$$

$$w_i = K_i / K_{\max}$$

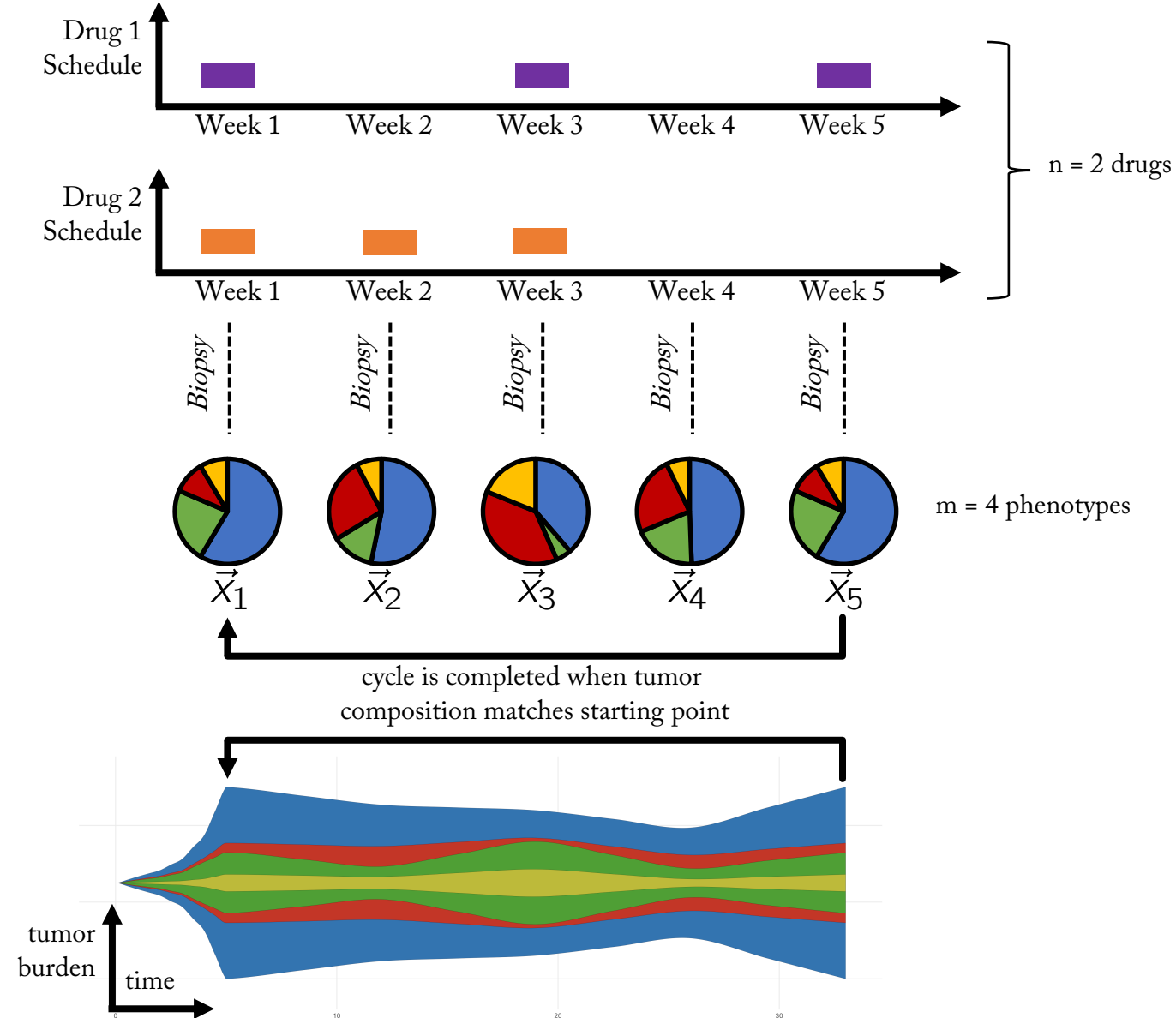
$$A = [1 - a_{ij}] = \begin{bmatrix} 1 - a_{11} & 1 - a_{12} & 1 - a_{13} \\ 1 - a_{21} & 1 - a_{22} & 1 - a_{23} \\ 1 - a_{31} & 1 - a_{32} & 1 - a_{33} \end{bmatrix}$$

Frequency-dependent Evolutionary Cycles



Key assumption:

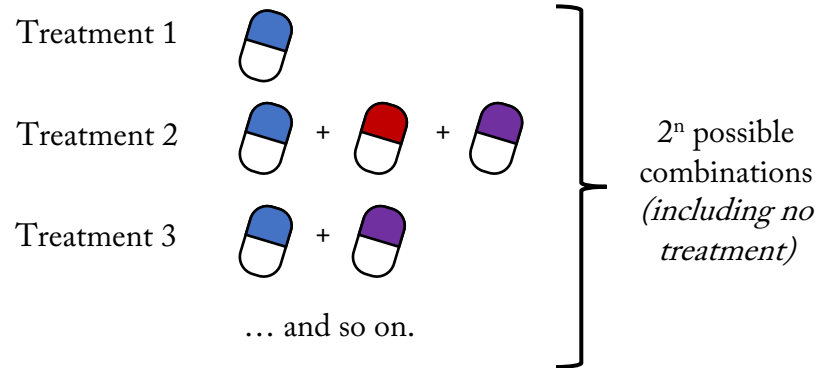
- since the goal of an adaptive therapy is to maintain a stable volume
- we can study frequency-dependent dynamics



What defines a treatment?



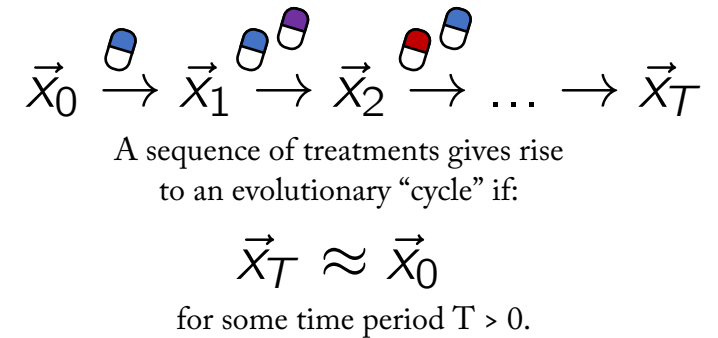
Combinations of
 n drugs:



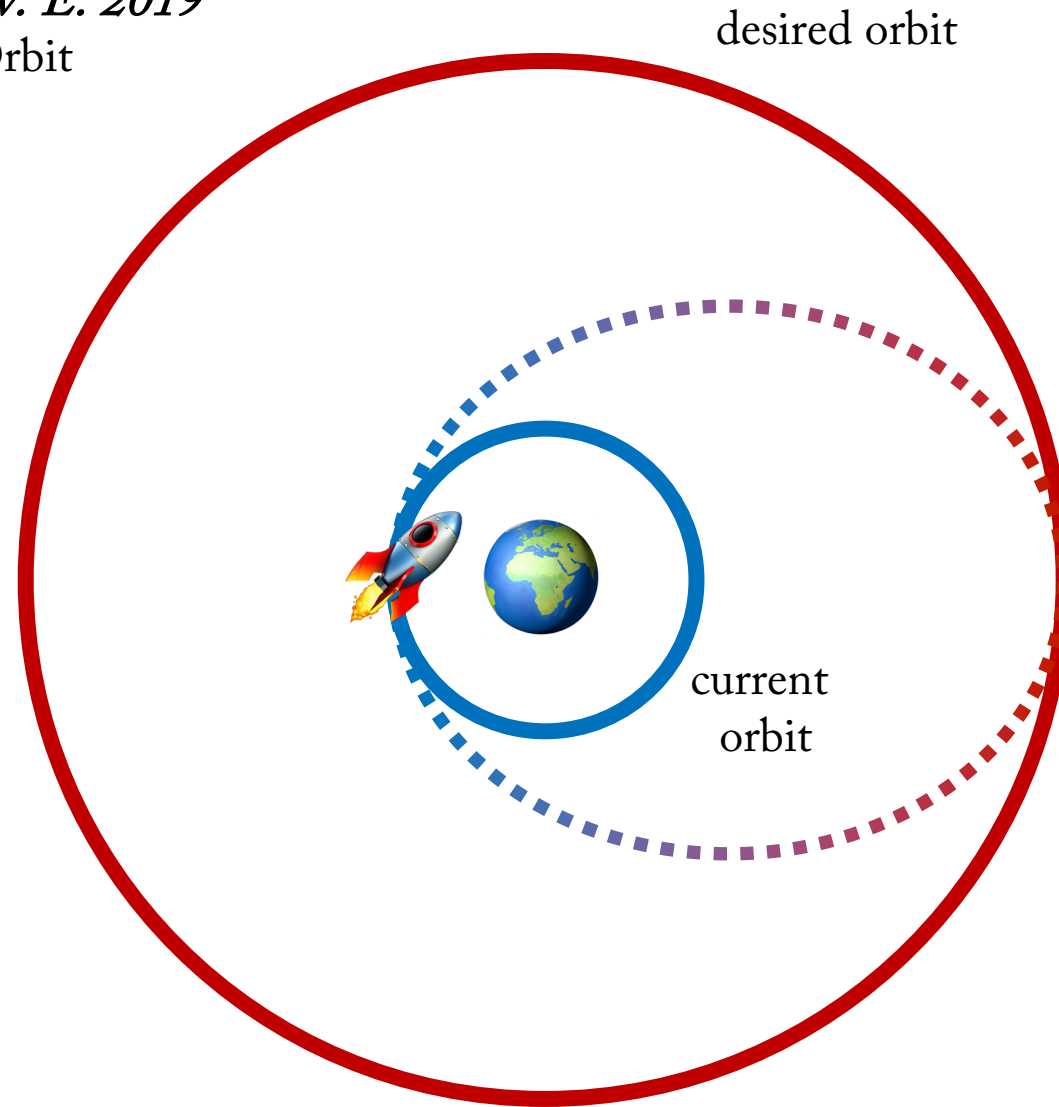
Controlling m cell types
(*genotype or phenotype*)

$$\vec{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Frequency-dependent cycles
of tumor evolution

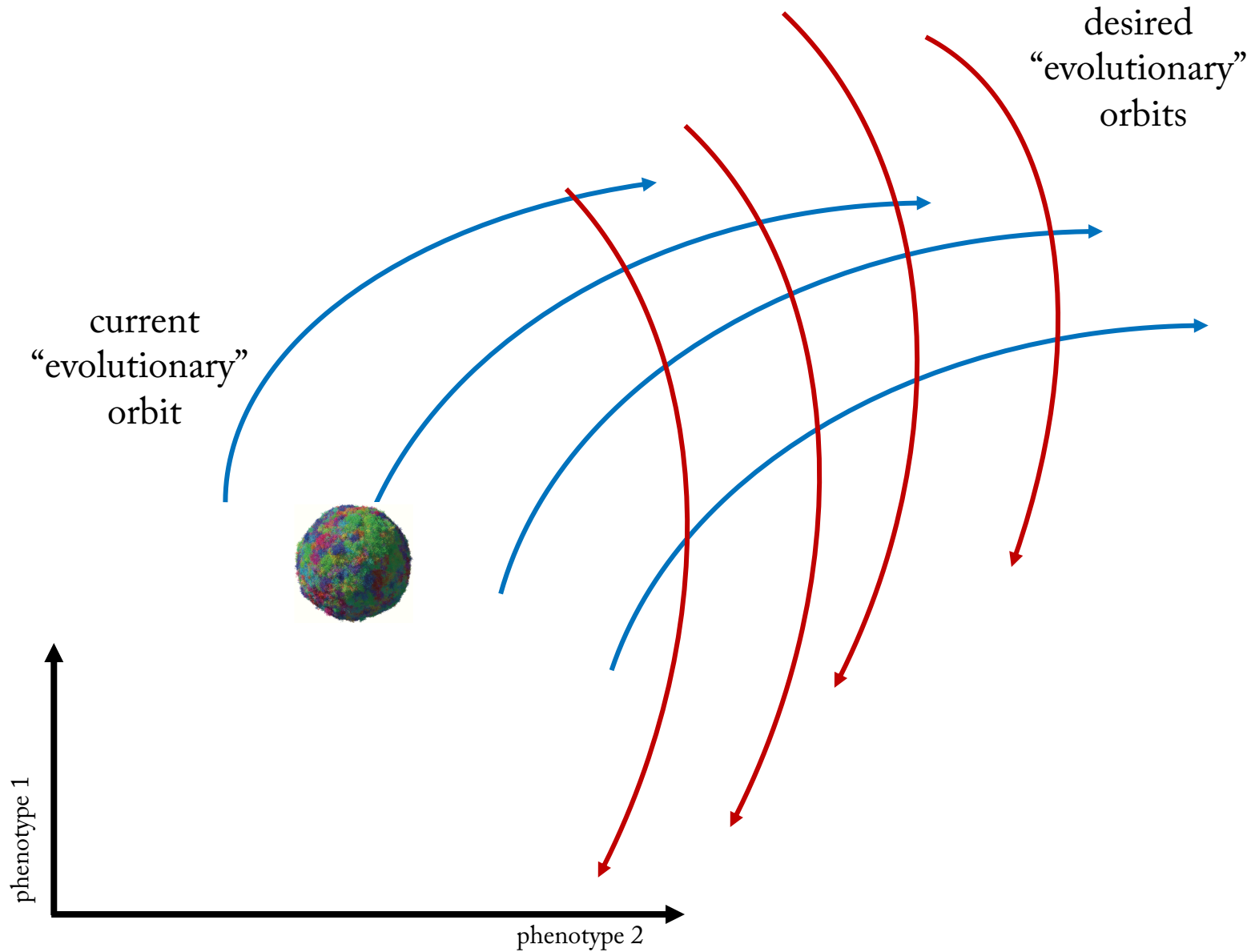


Newton et. al. Phys. Rev. E. 2019
- Hohmann Transfer Orbit

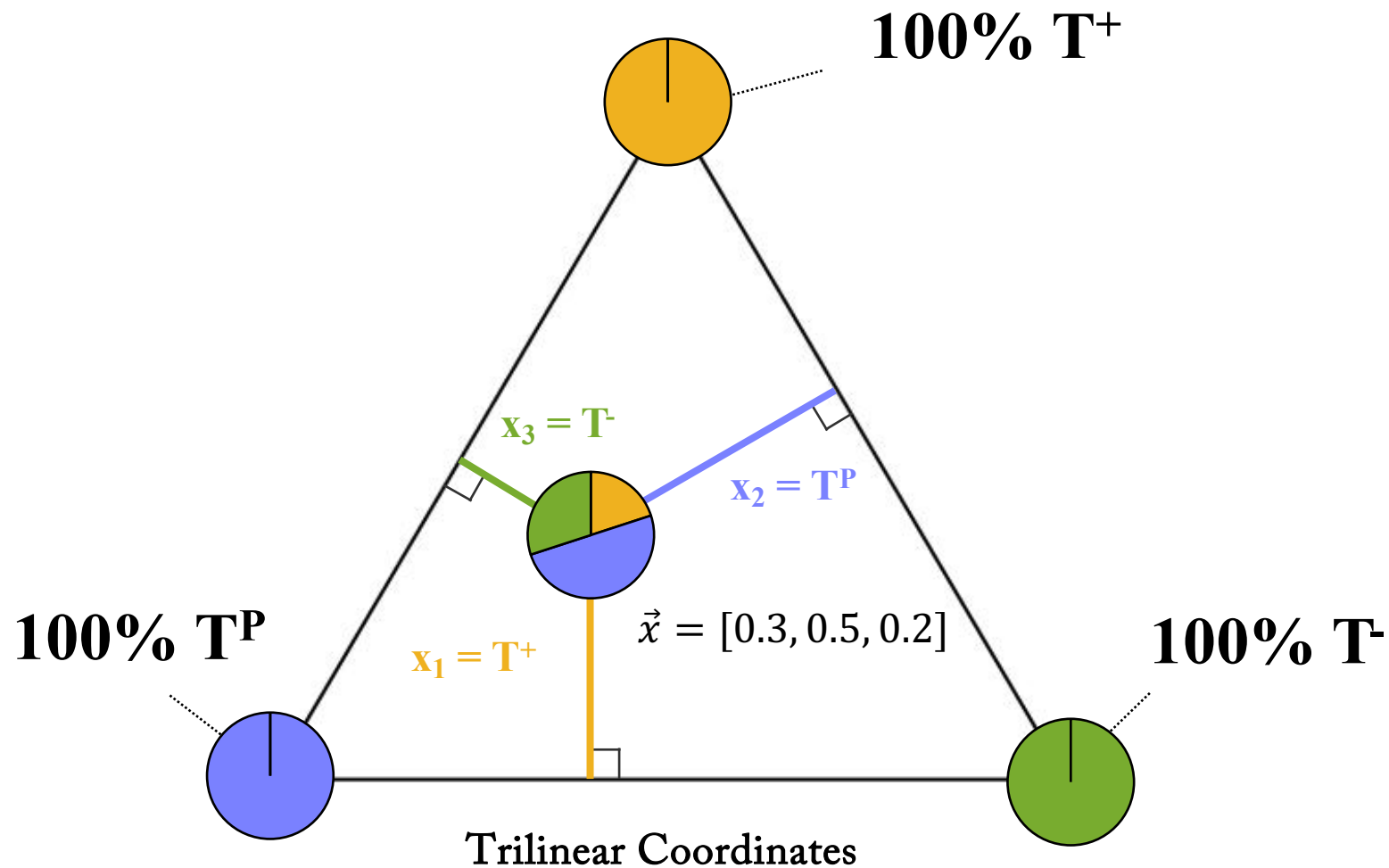


1. Current orbit is fixed & known
2. Desired orbit is fixed & known
3. The connecting orbit is tangent to current orbit and intersects desired orbit

Drawing inspiration from orbital mechanics



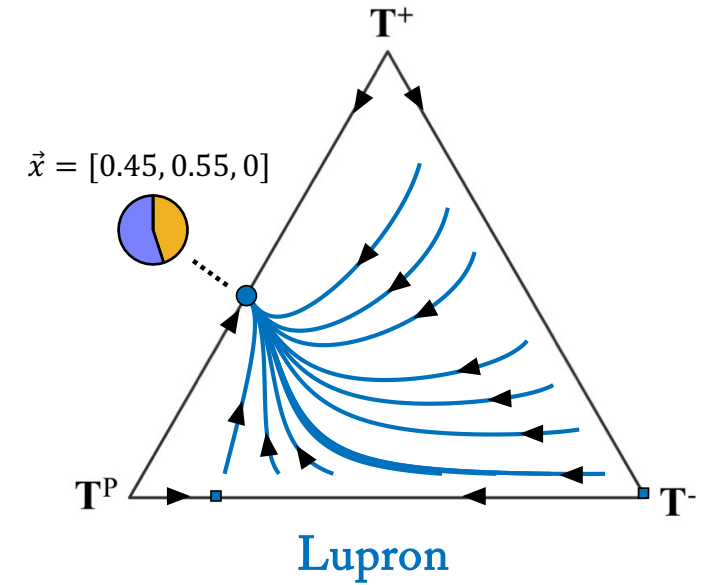
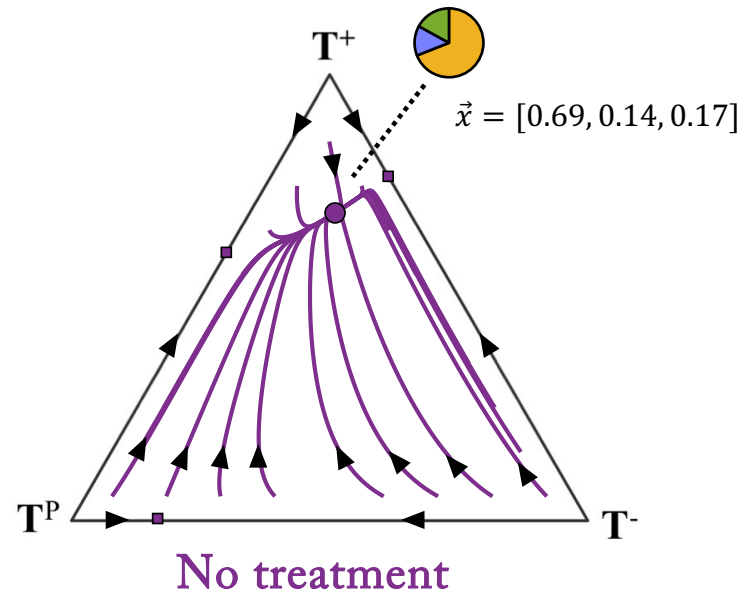
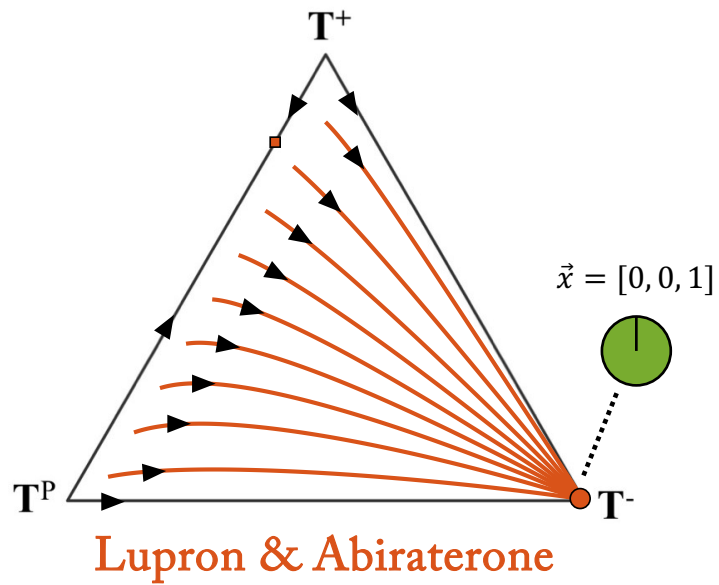
1. Current orbit is fixed & known
2. Desired orbit is fixed & known
3. The connecting orbit is tangent to current orbit and intersects desired orbit



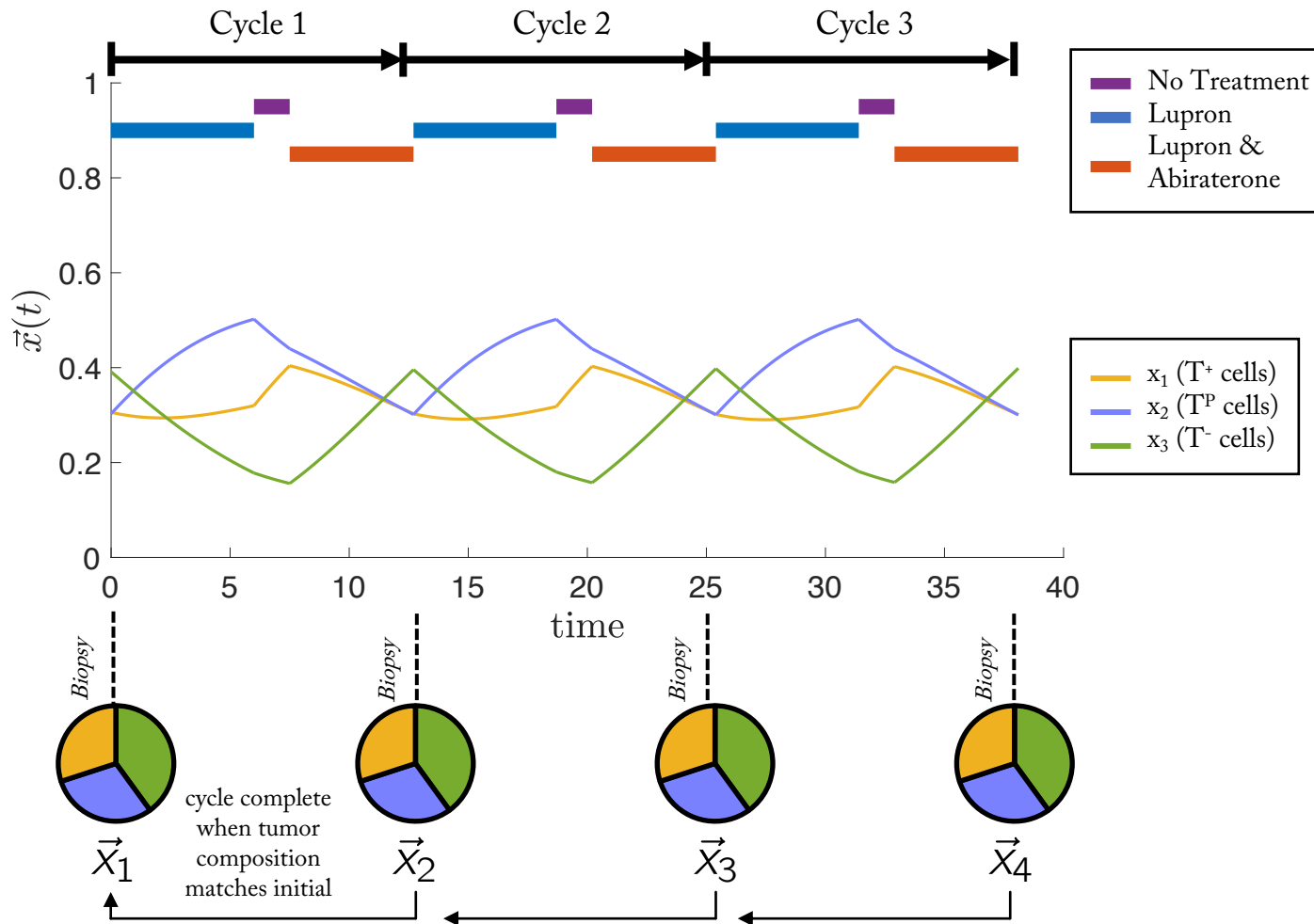
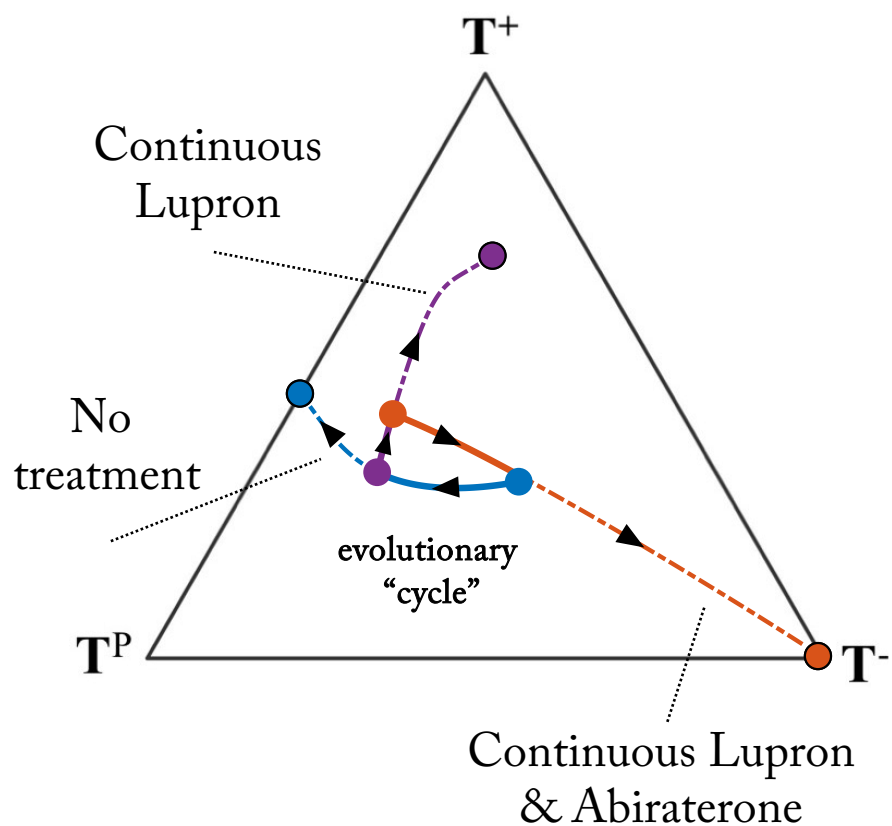
Trilinear Simplex

- The state space of all possible states of three populations
- $x_1 + x_2 + x_3 = 1$

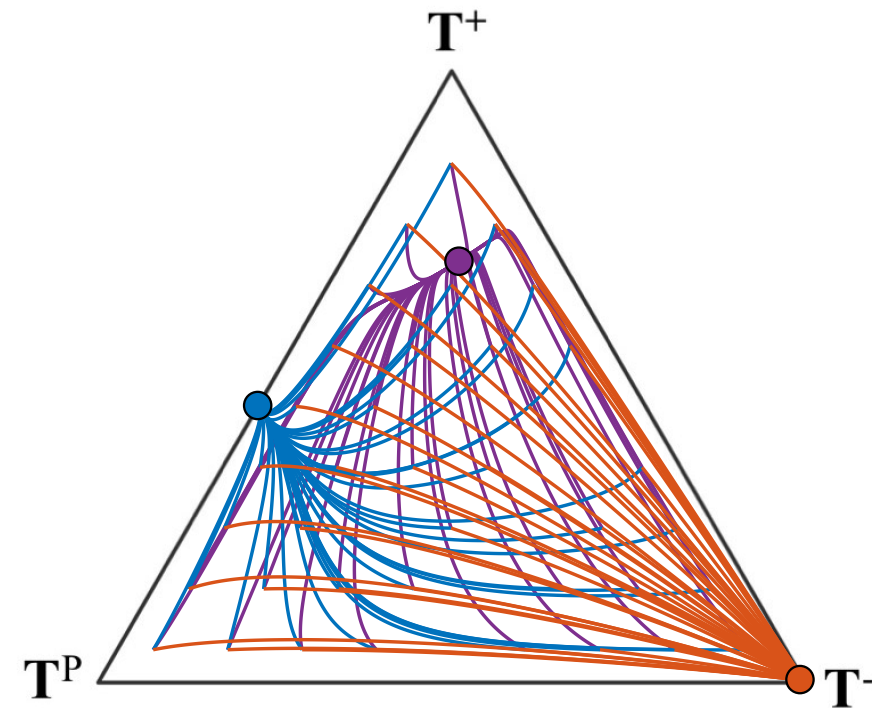
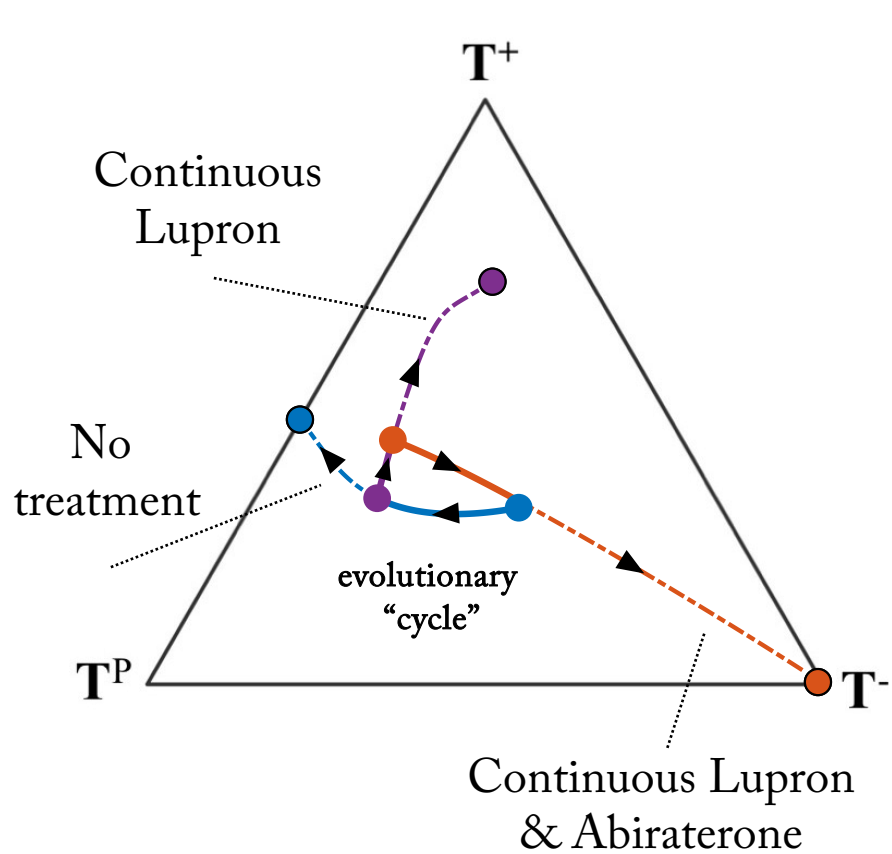
Frequency-dependent treatment dynamics



Frequency-dependent treatment dynamics



Frequency-dependent treatment dynamics

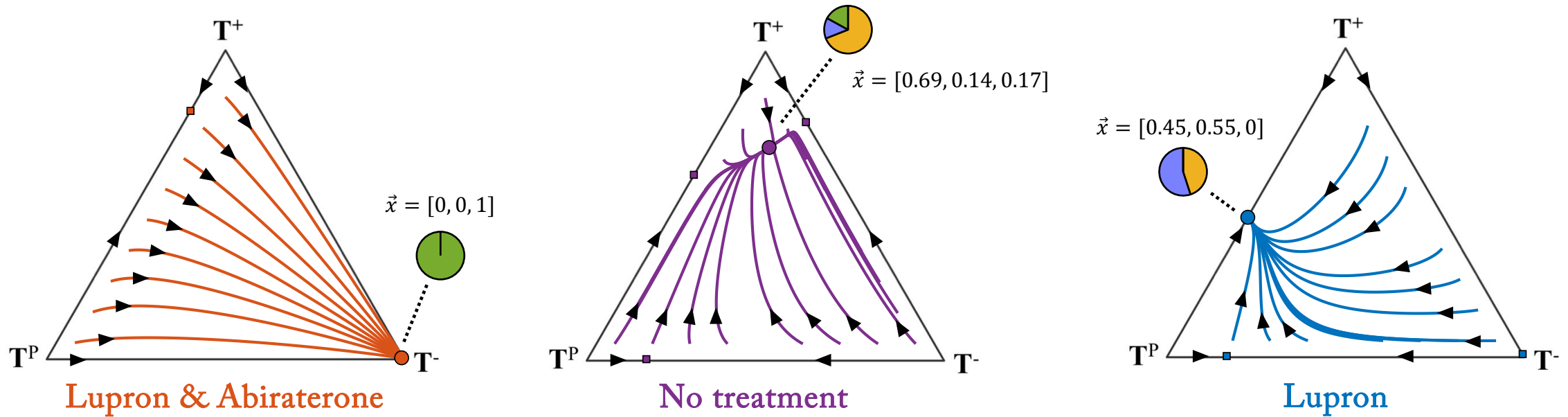


3. Evolutionary search space

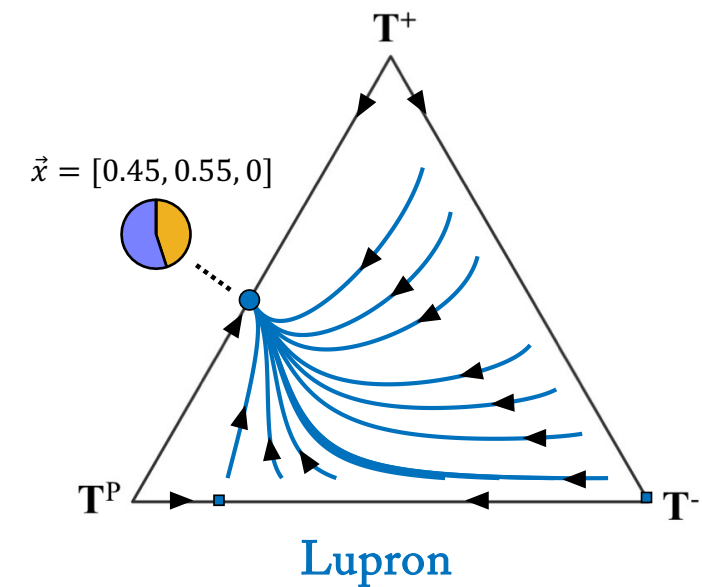
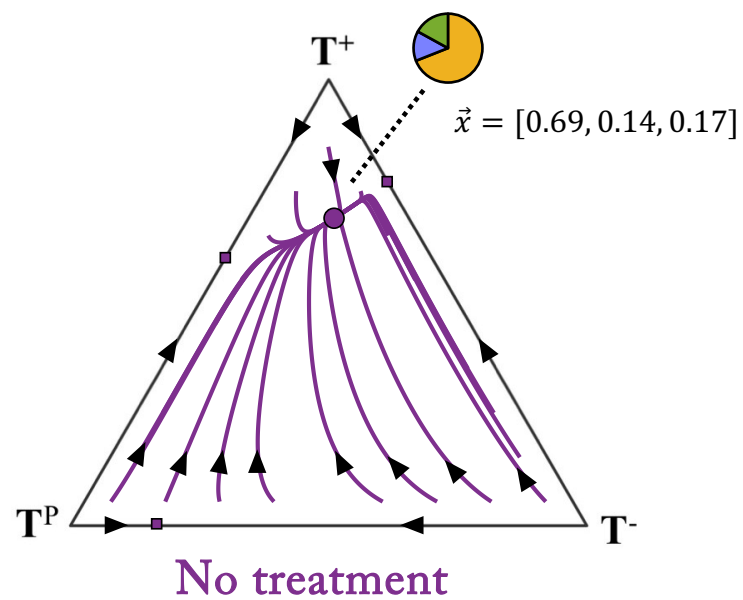
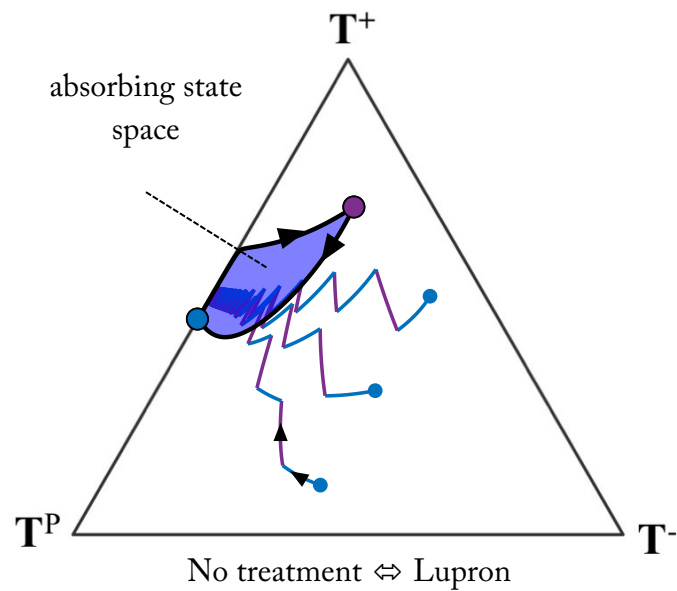
" Most discoveries even today are a combination of serendipity and of searching."

- Siddhartha Mukherjee -

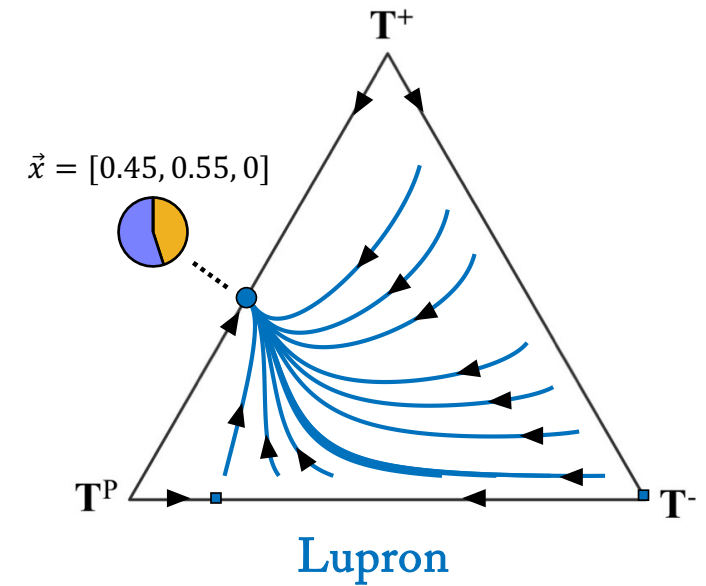
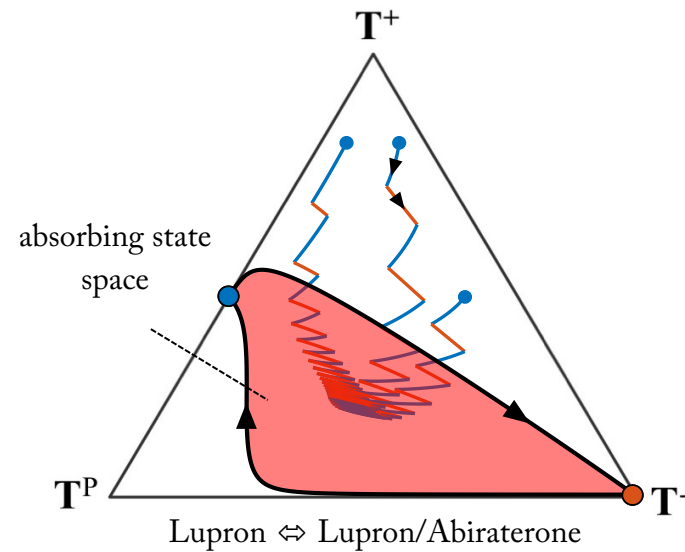
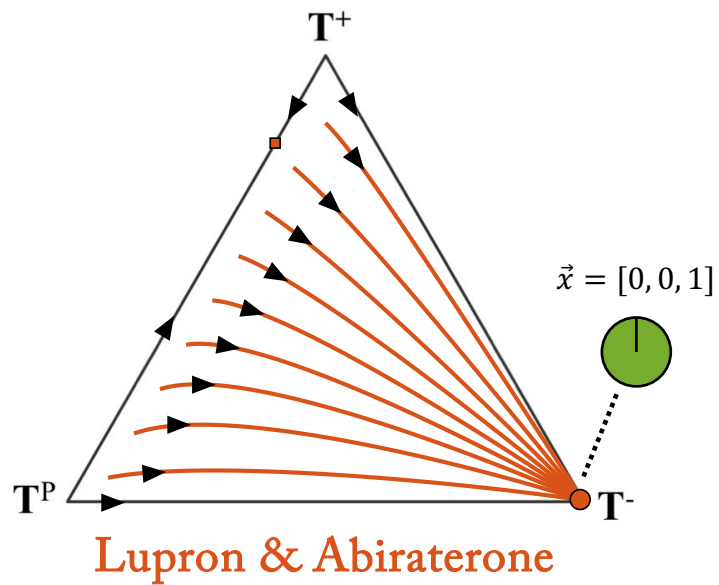
Pairwise treatment combinations



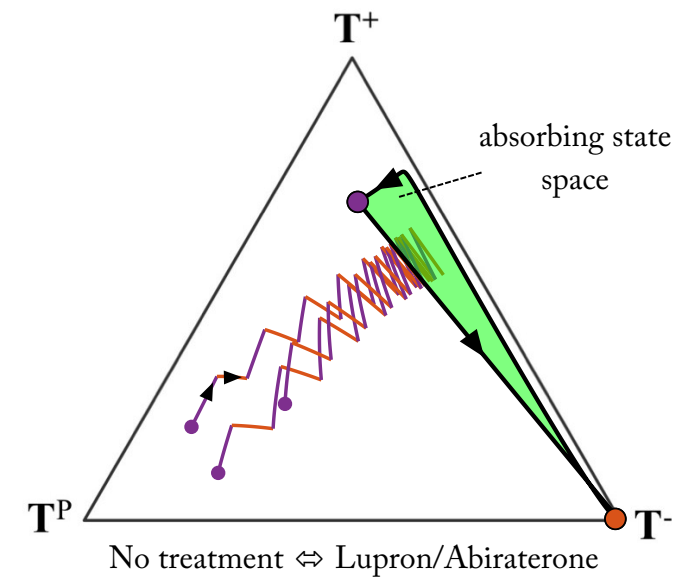
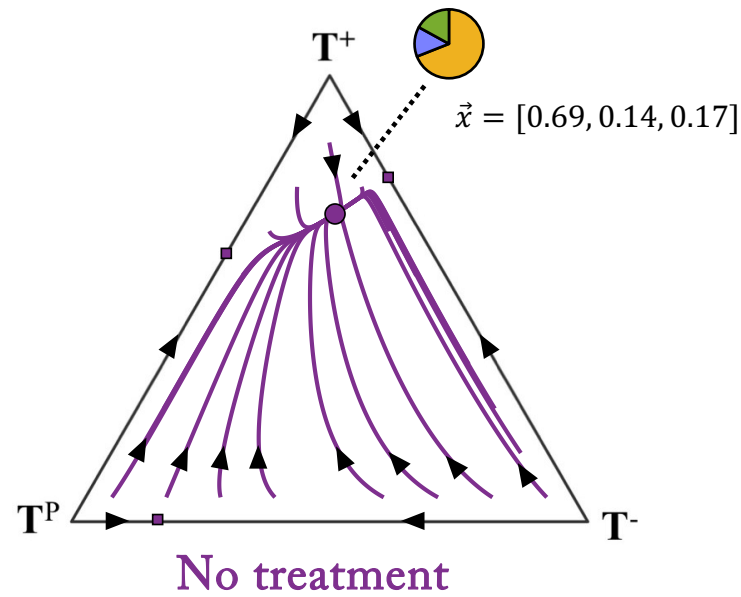
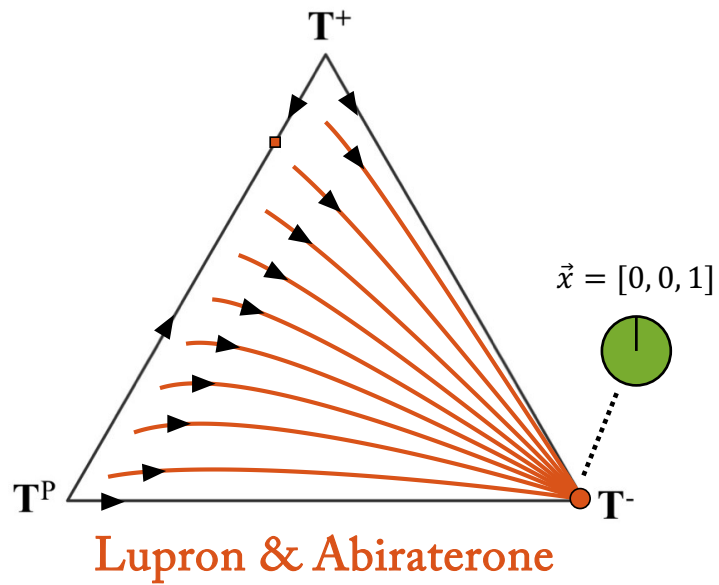
Evolutionary Search Space



Evolutionary Search Space



Evolutionary Search Space

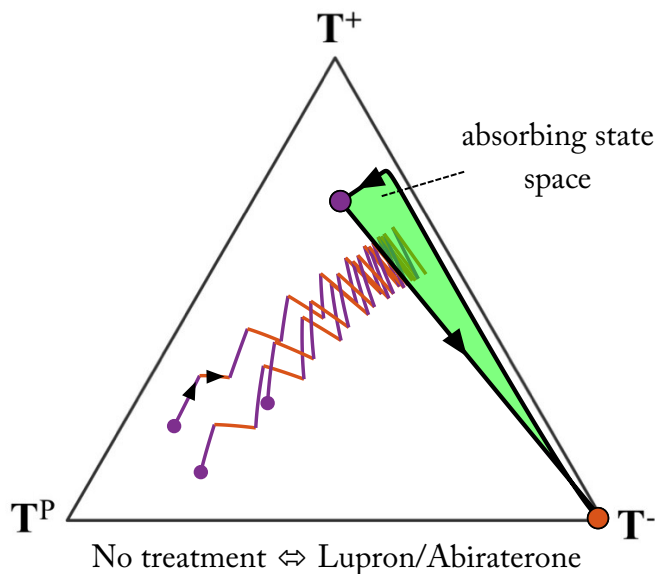
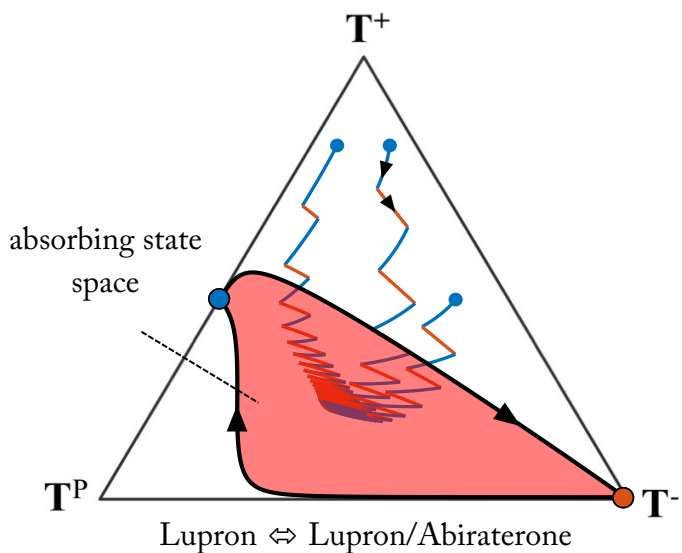
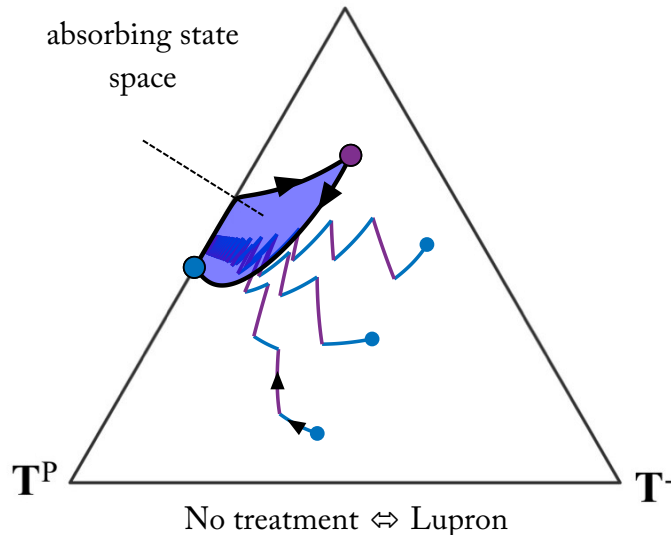


Evolutionary Search Space



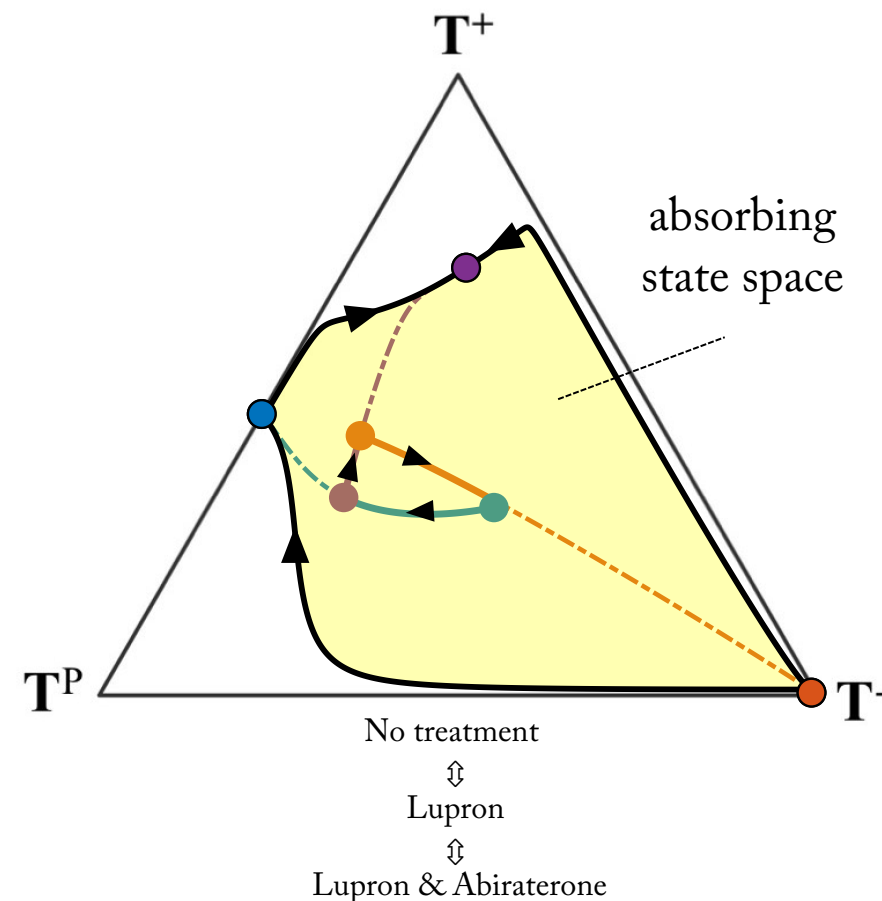
Key observations:

- All possible drug sequences tend toward absorbing area
- Various drug combinations have varied size of absorbing space
- The longer time cycled, closer to absorbing space
- No treatment is an evolutionary process



3 treatments

- Vastly expands absorbing state space
- “Orthogonal” drugs are desirable



Payoff assumptions

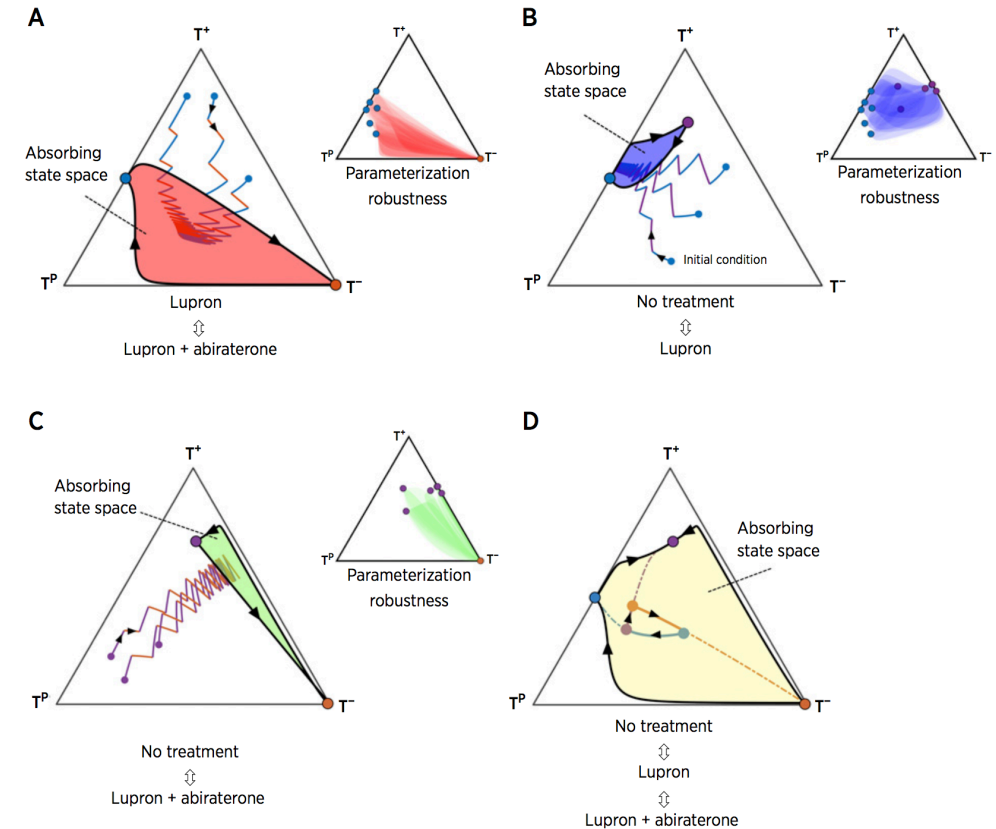


No treatment

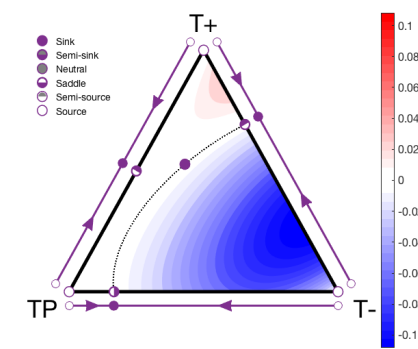
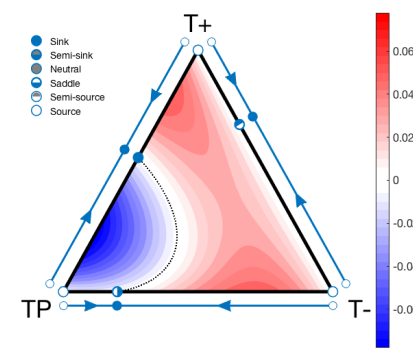
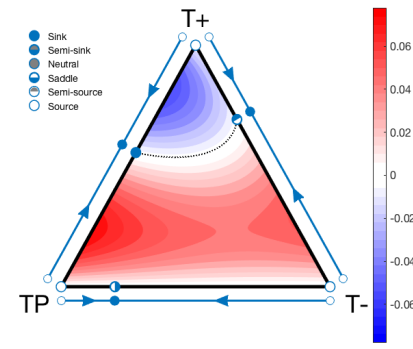
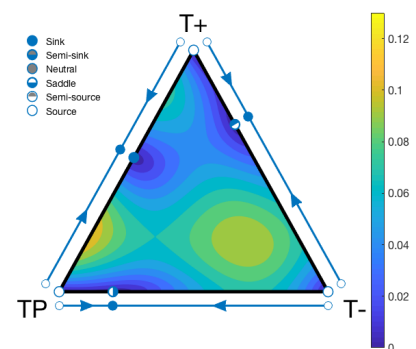
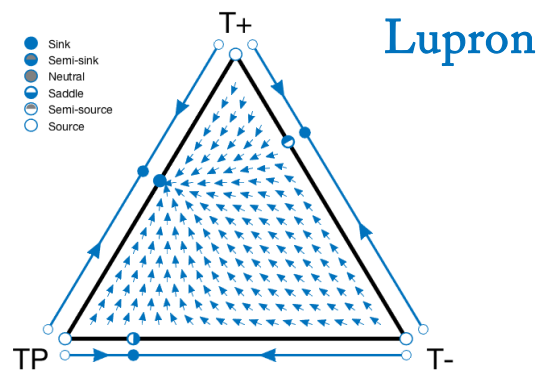
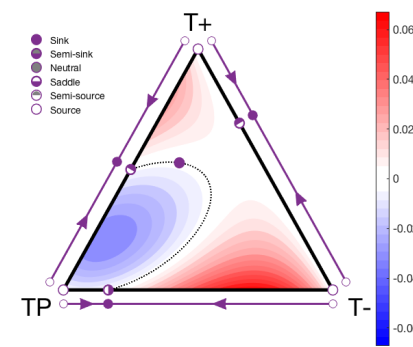
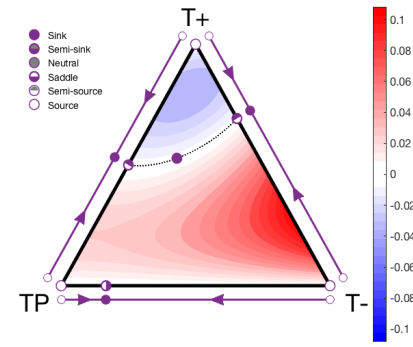
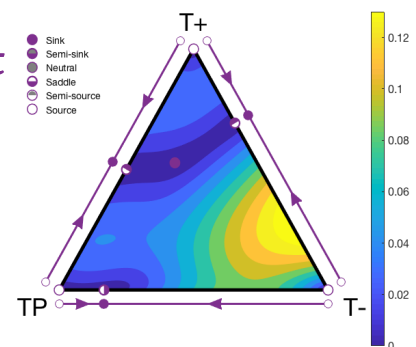
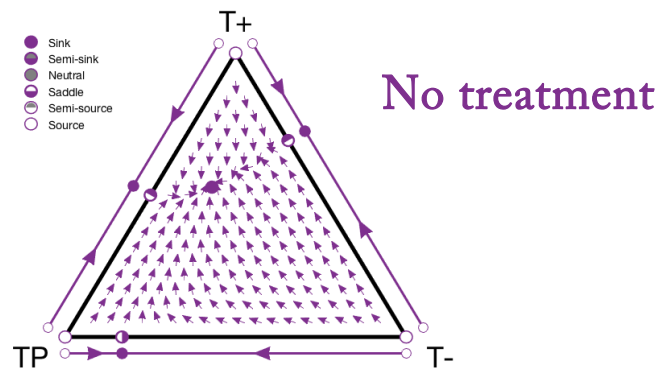
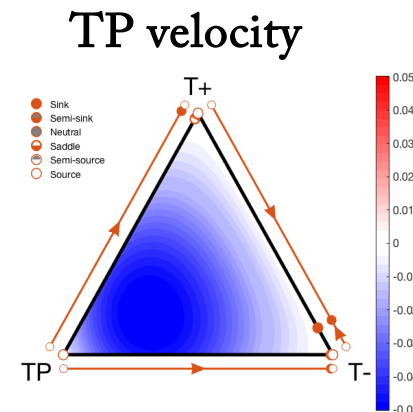
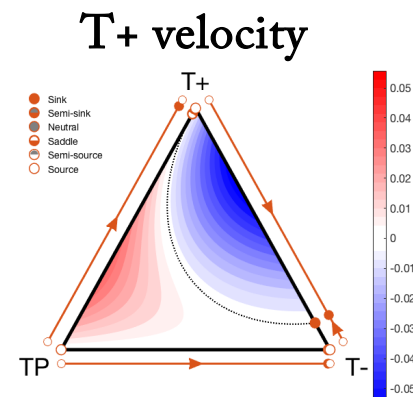
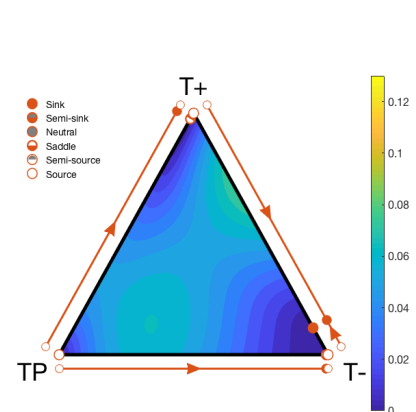
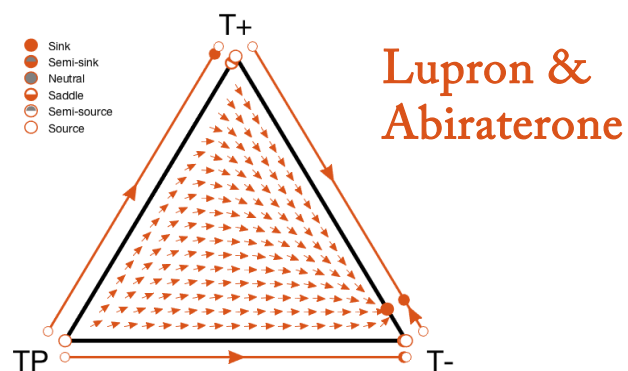
$c < e$	T^- cells have a higher fitness than T^P cells when interacting with many T^+ , especially in low vasculature regions. Testosterone production by T^P production comes at some cost to provide public good to both self (T^P) and neighbor (T^+). Both c and e should decrease in the pre-treatment condition
$a > f$	T^+ cells have a higher fitness than T^- cells when interacting with many T^P , receiving advantage from the public good. The parameter a should increase in the pre-treatment condition f slightly decrease
$b > d$	T^+ cells have a higher fitness than T^P cells when interacting with many T^- because there is lack of spatial competition near vasculature for T^+ cells as testosterone is not being used
$a < b$	T^+ cells have a higher fitness competing with T^- over competition with T^P
$c < d$	Similarly, T^P cells have a higher fitness competing with T^- over competition with T^+
$e > f$	T^- cells have less competition for space in a tumor with mostly T^+ than with mostly T^P . The parameter f should decrease in the pre-treatment condition slightly

Lupron & Abiraterone

$c > e$	T^P cells have a higher fitness than T^- cells when interacting with few T^+ (absence of competition)
$a > f$	Interacting with mostly T^P cells, T^+ gains from the public good and from the new available space in low vasculature regions
$b < d$	Interacting with mostly T^- cells, T^P cells see little competition near vasculature. Payoffs to T^+ cells, b , may be small or zero
$a > b = 0$	T^+ cells need the T^P cells to succeed in the absence of systemic testosterone
$c > d$	c is likely the largest parameter as T^P cells have the highest fitness in a mostly T^- tumor without systemic testosterone
$e > f$	Again, T^P cells outcompete T^- cells in the absence of systemic testosterone



Velocity of frequency-dependent dynamics



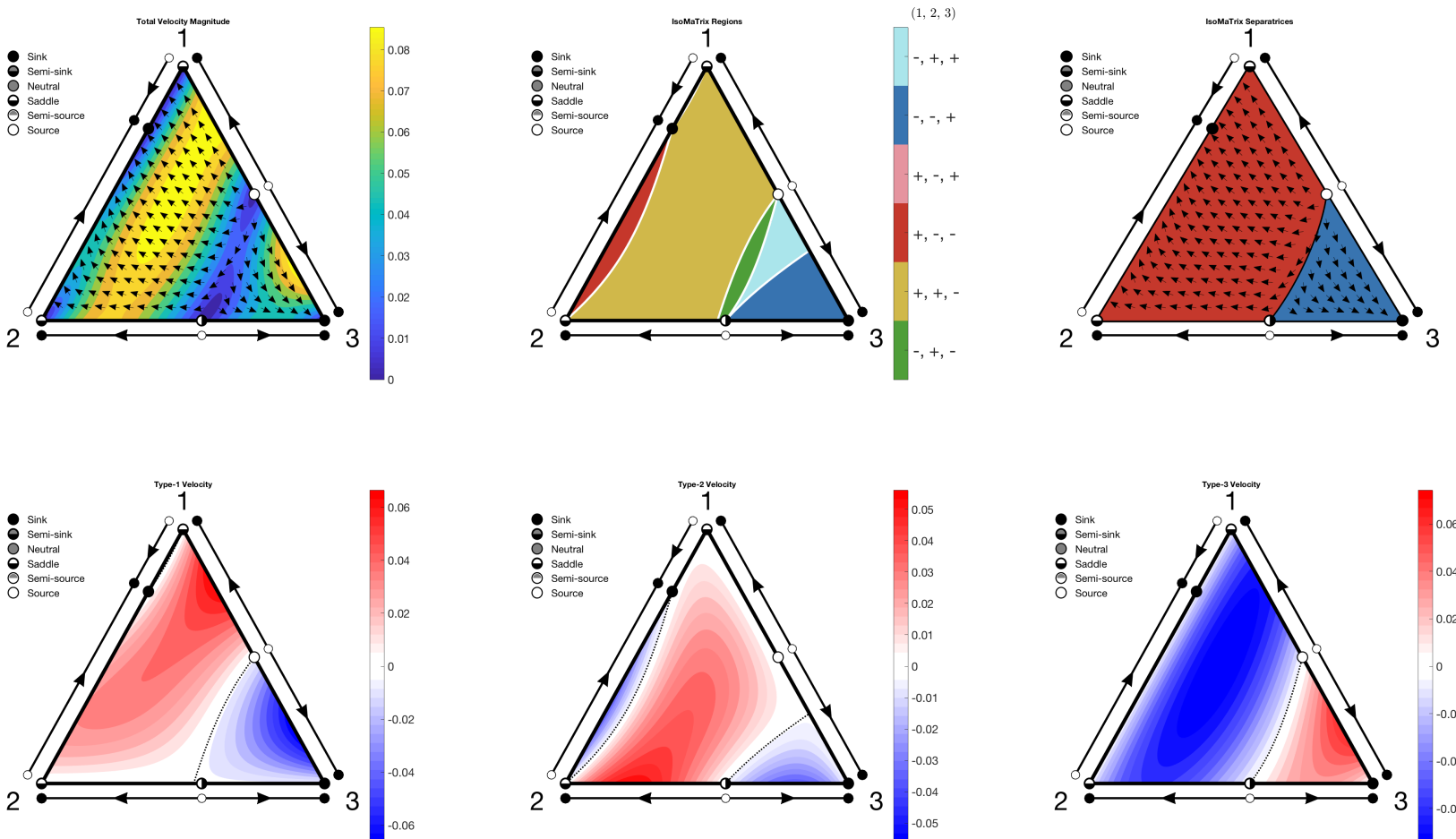
4. IsoMaTrix

A tool for visualizing matrix games

"The best investment is in the tools of own's own trade."

- Benjamin Franklin -

IsoMaTriX: visualizing Isoclines of Matrix games



Download at:

- github.com/MathOnco/isomatrix

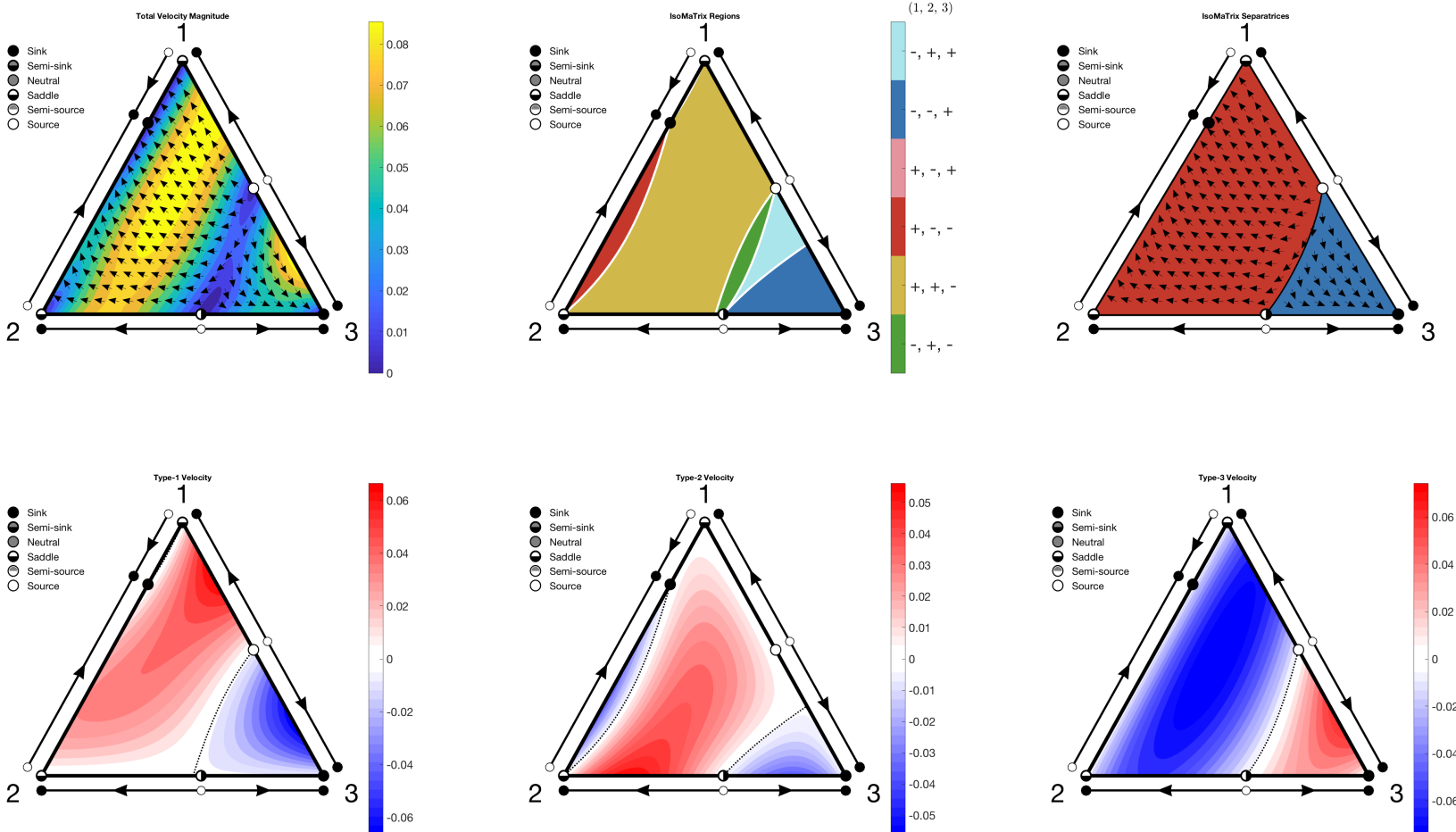
Joint work with:

- Yongqian Ma (Univ. Southern California)
- Artem Kaznatcheev (Oxford Univ)
- Alexander Anderson (Moffitt)

How to use:

- 6 graphs with one command: `isomatrix(A)`

IsoMaTrix: visualizing Isoclines of Matrix games



1 IsoMaTrix (MATLAB)

- 1.1 `isomatrix(A)`
- 1.2 `isomatrix_fixedpoint(A,index)`
- 1.3 `isomatrix_quiver(A)`
- 1.4 `isomatrix_isocline(A,id)`
- 1.5 `isomatrix_trajectory(A,x0,tF)`
- 1.6 `isomatrix_velocity(A,id)`
- 1.7 `isomatrix_fitness(A,id)`
- 1.8 `isomatrix_region(A)`
- 1.9 `isomatrix_surface(A,id)`
- 1.10 `isomatrix_separatrix(A)`

2 IsoMaTrix Helper Functions (MATLAB)

- 2.1 `Coordinate transformations`
- 2.2 `replicator(t,x,A)`
- 2.3 `line_plot(A,x0,tF)`
- 2.4 `add_labels(string_array)`
- 2.5 `add_gridlines(gridlines)`
- 2.6 `pairwise_fixedpoint(A)`
- 2.7 `hessian(x,A)`
- 2.8 `A_subset(A,types)`
- 2.9 `Ohtsuki_Nowak_transform(A,k,rule)`

3 HAL integration with IsoMaTrix (Java)

- 3.1 `Setting up Integrated Development Environment`
- 3.2 `HALMatrixGame2D and HALMatrixGame3D`
- 3.3 `Fitness Neighborhood`
- 3.4 `Deterministic or Stochastic Updating`
- 3.5 `Population Update Fraction`
- 3.6 `SingleSimulation(int timesteps)`
- 3.7 `MeshGrid(int timesteps, int nSims)`

4 Visualizing HALMatrixGames using IsoMaTrix

- 4.1 `HAL_isomatrix()`
- 4.2 `HAL_isomatrix_trajectory(color)`
- 4.3 `HAL_isomatrix_quiver(uncertainty_boolean)`
- 4.4 `HAL_isomatrix_velocity(id)`
- 4.5 `HAL_isomatrix_region()`
- 4.6 `HAL_isomatrix_uncertainty(id)`



Integrated Mathematical Oncology

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